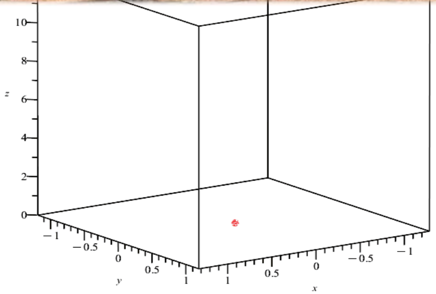

3D space trajectories and beyond: abstract art creation with 3D printing

JERUSALEM COLLEGE OF TECHNOLOGY
LEV ACADEMIC CENTER

JOHANNES KEPLER UNIVERSITY LINZ

ADG 2023
14th International Conference on Automated Deduction in Geometry
Belgrade, Serbia
September 20-22, 2023

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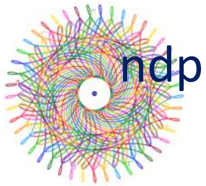


Motivations

Newspapers and TV news are full of reports about launching satellites to space orbiting objects in space from for example the International Space Station (ISS), the Chinese space station.

Projects such as Mars exploration and the Artemis project aim to establish a permanent human presence on the moon.

In August 2020, for instance, three spacecrafts have been launched towards Mars and NASA enables the public to register for sending their names on a probe, which will launch in 2024 and will arrive at Encelade, an icy moon of Jupiter, in 2030.

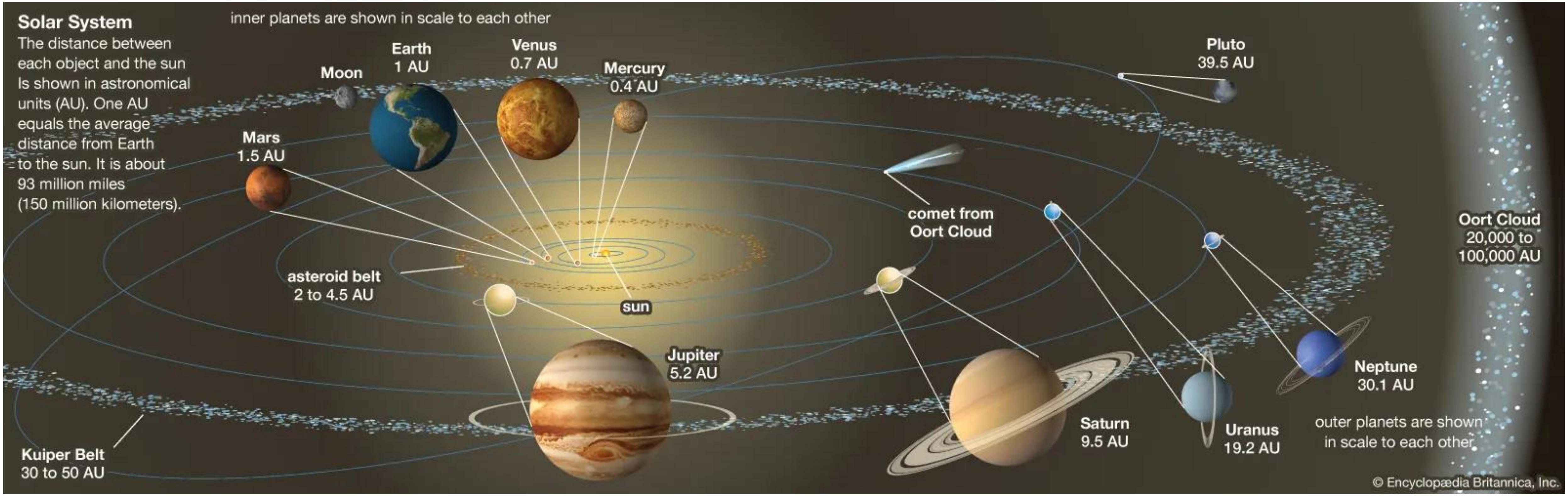
With such an ubiquitous topic, interest in students is raised and some ask a lot of questions.

Some of them ask about spacecrafts, many of them wish to understand the trajectories.

S T E A M
Science Technology Engineering Arts Mathematics

The Solar System

For some incitations to astronomy and related topics, refer also e.g. to Lila Korinova's talk earlier today (using AR and smartphones)



Because of the wide range of orbits' radii, it is impossible to represent all the system respecting the true proportions.
In what follows, we consider pairs of neighboring planets.

Modelling: standard procedure

Mathematical modelling is characterized through its interplay of reality and mathematics. It offers a way to integrate references to reality into the classroom and shows students where in everyday life their mathematical knowledge can be applied."

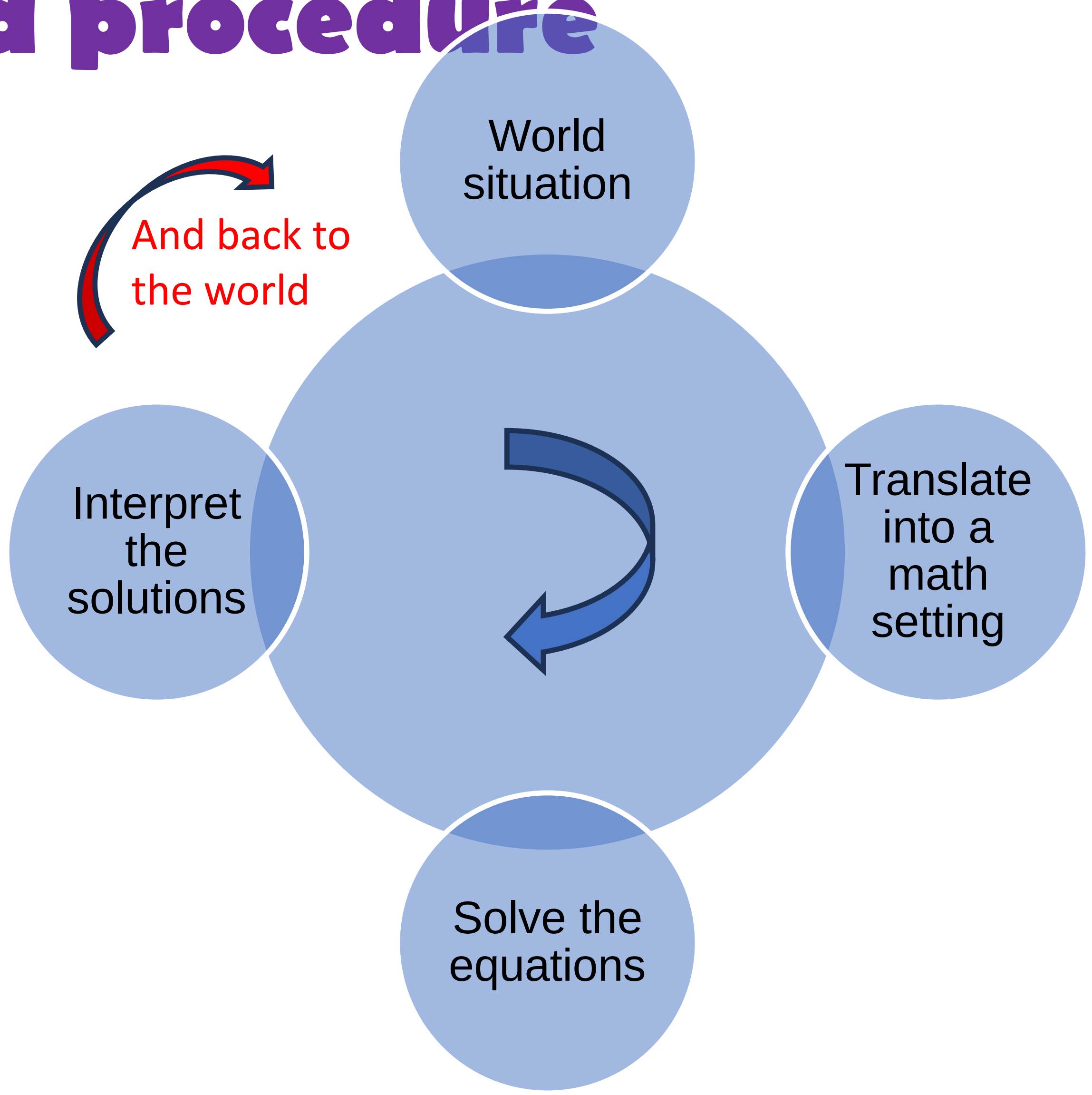
The process:

a. A real-world problem is given and analyzed.

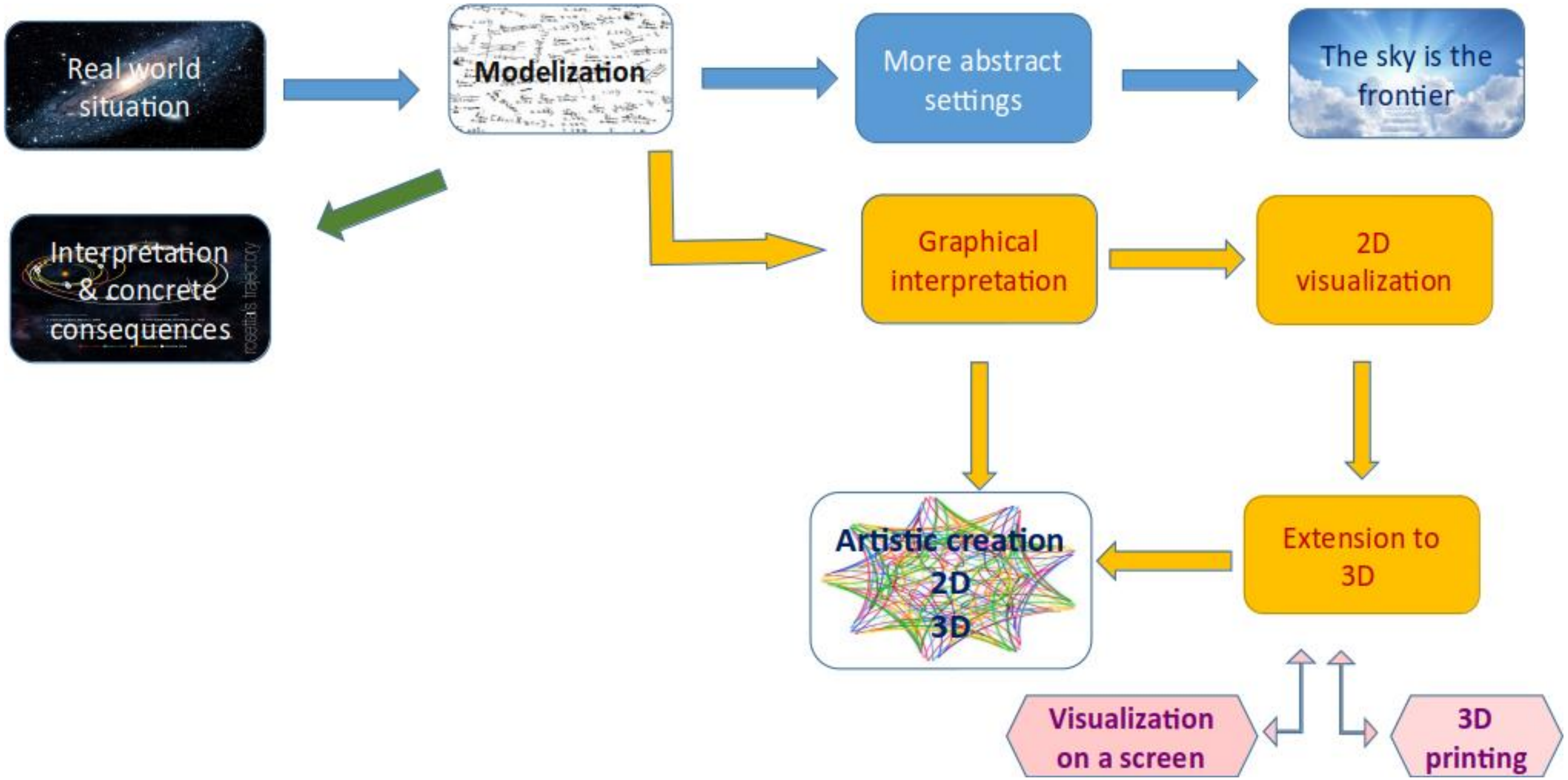
b. It is translated into a mathematical setting (Descartes claimed that every problem has to be transformed into a system of equations).

c. The mathematical problem is solved.

d. The solutions have to be interpreted and validated with respect to reality.



From modelling towards other directions

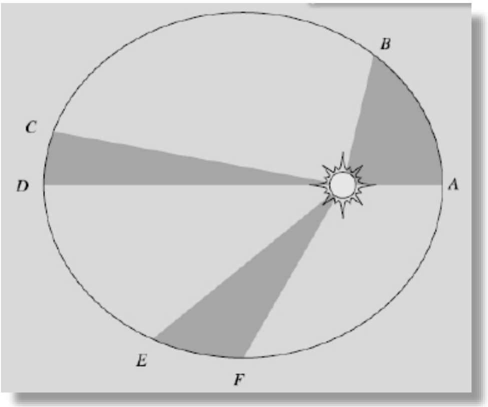


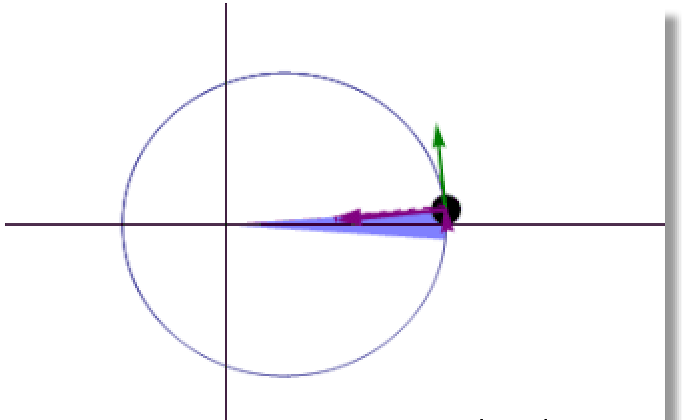
Kepler laws

We illustrate here this law

Oh, thanks!!

- **First law:** The orbit of every planet is an ellipse with the Sun at one of the two foci.
- **Second law:** A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.

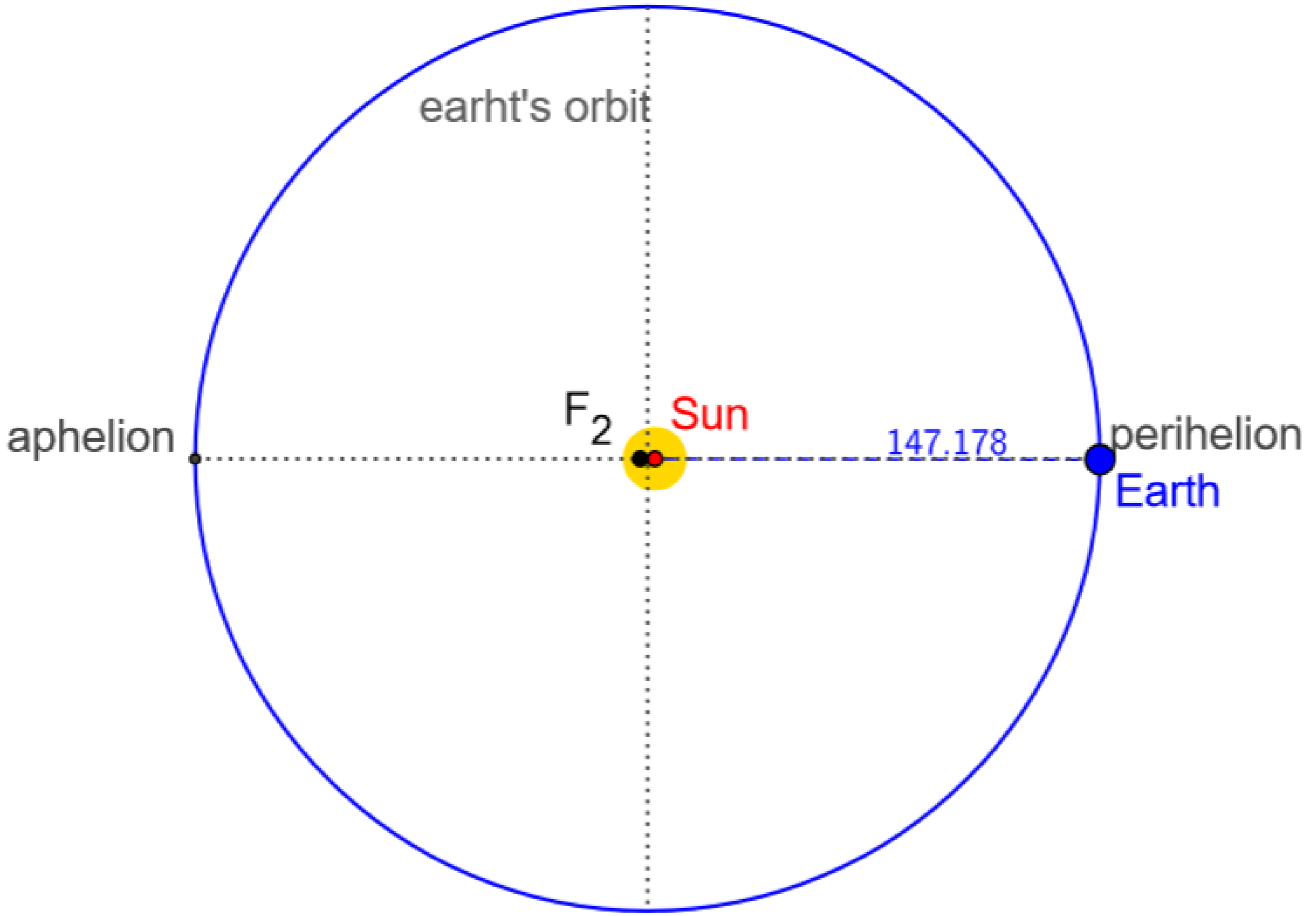


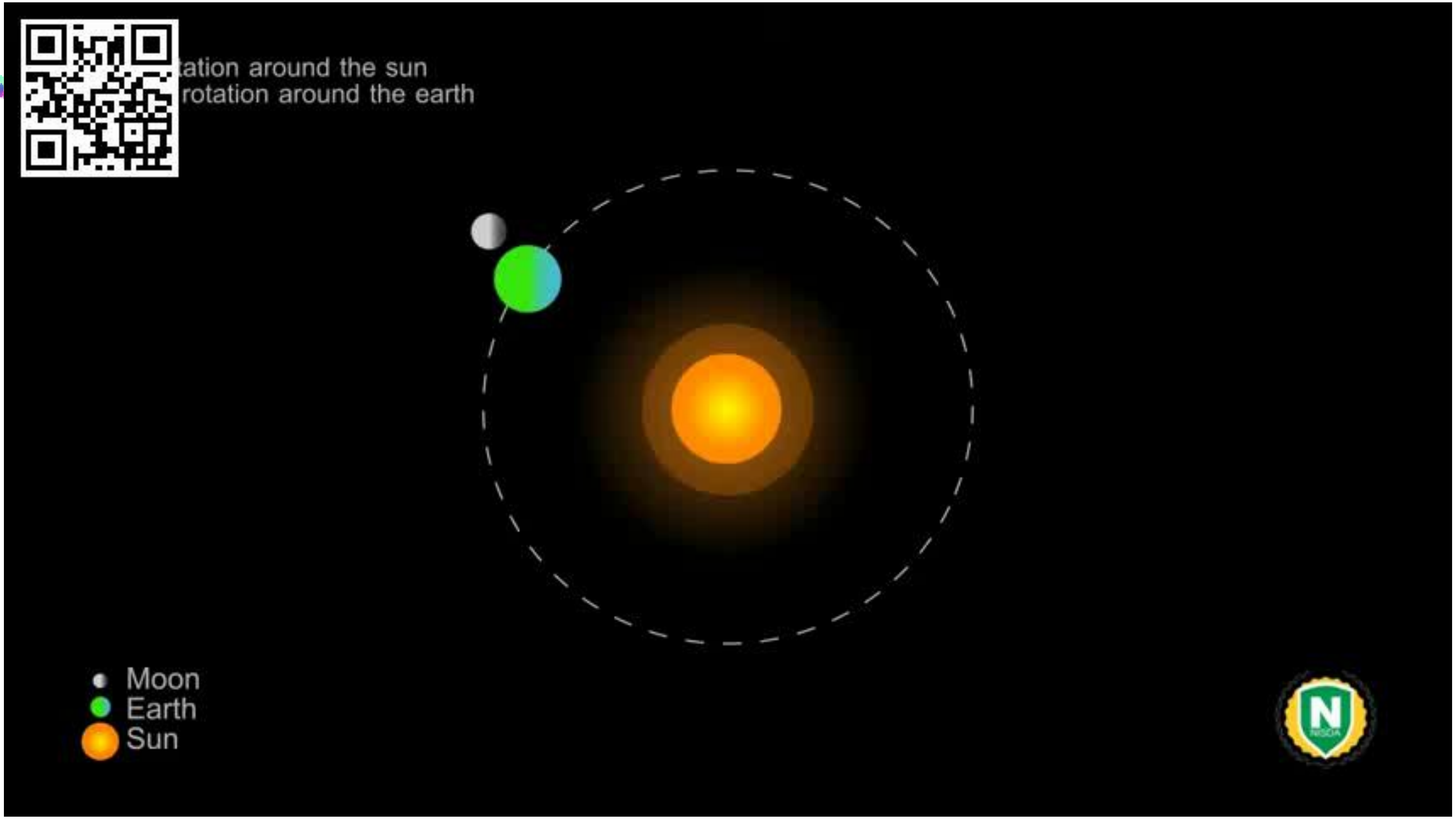


Source: wikipedia

- **Third law:** The ratio of the square of an object's orbital period with the cube of the semi-major axis of its orbit is the same for all objects orbiting the same primary.

Why circular models?

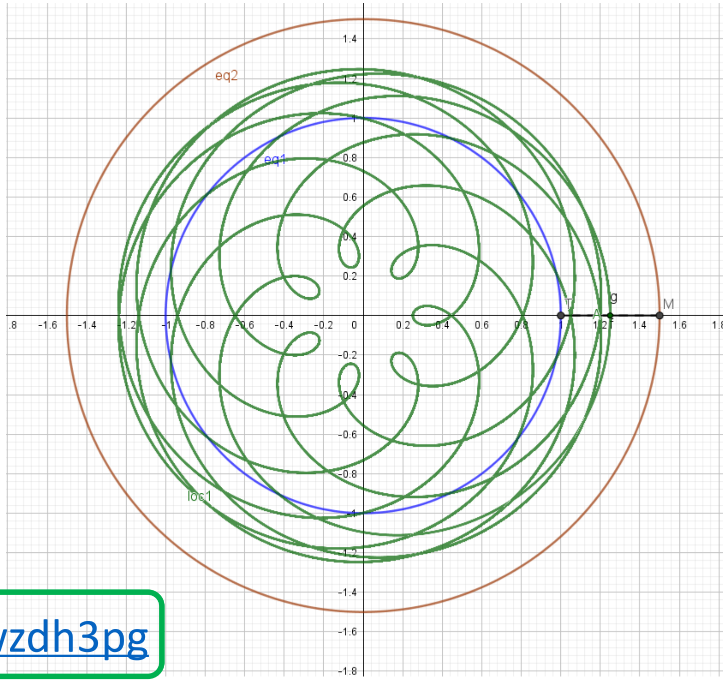


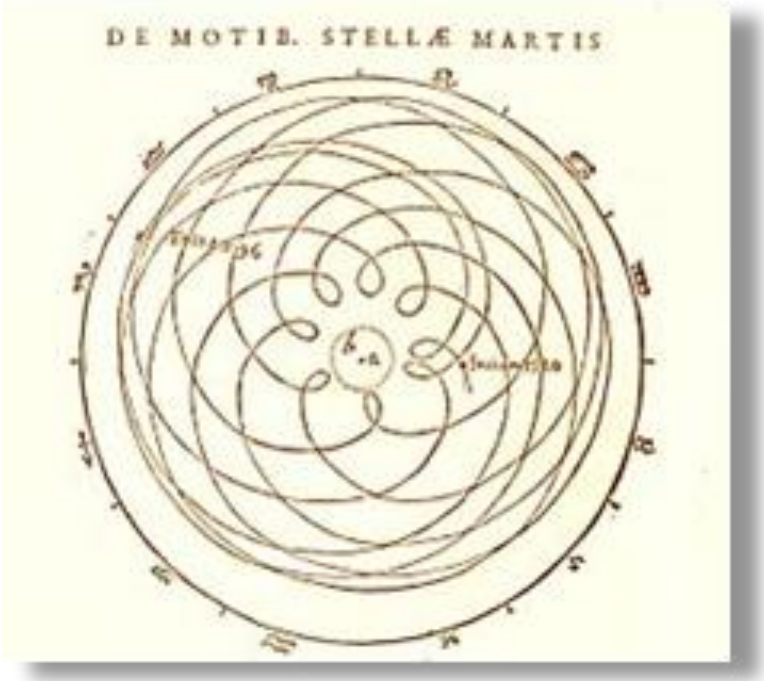


Animation: https://youtu.be/W47Wa7onrIQ

Space curves, orbits and Kepler's drawing

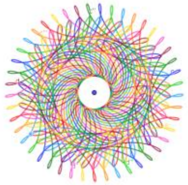
Th. Dana-Picard and S. Hershkovitz (2023): From Space to Maths And to Arts: Virtual Art in Space with Planetary Orbits, to appear in Electronic Journal of Mathematics & Technology.





https://www.geogebra.org/m/awzdh3pg

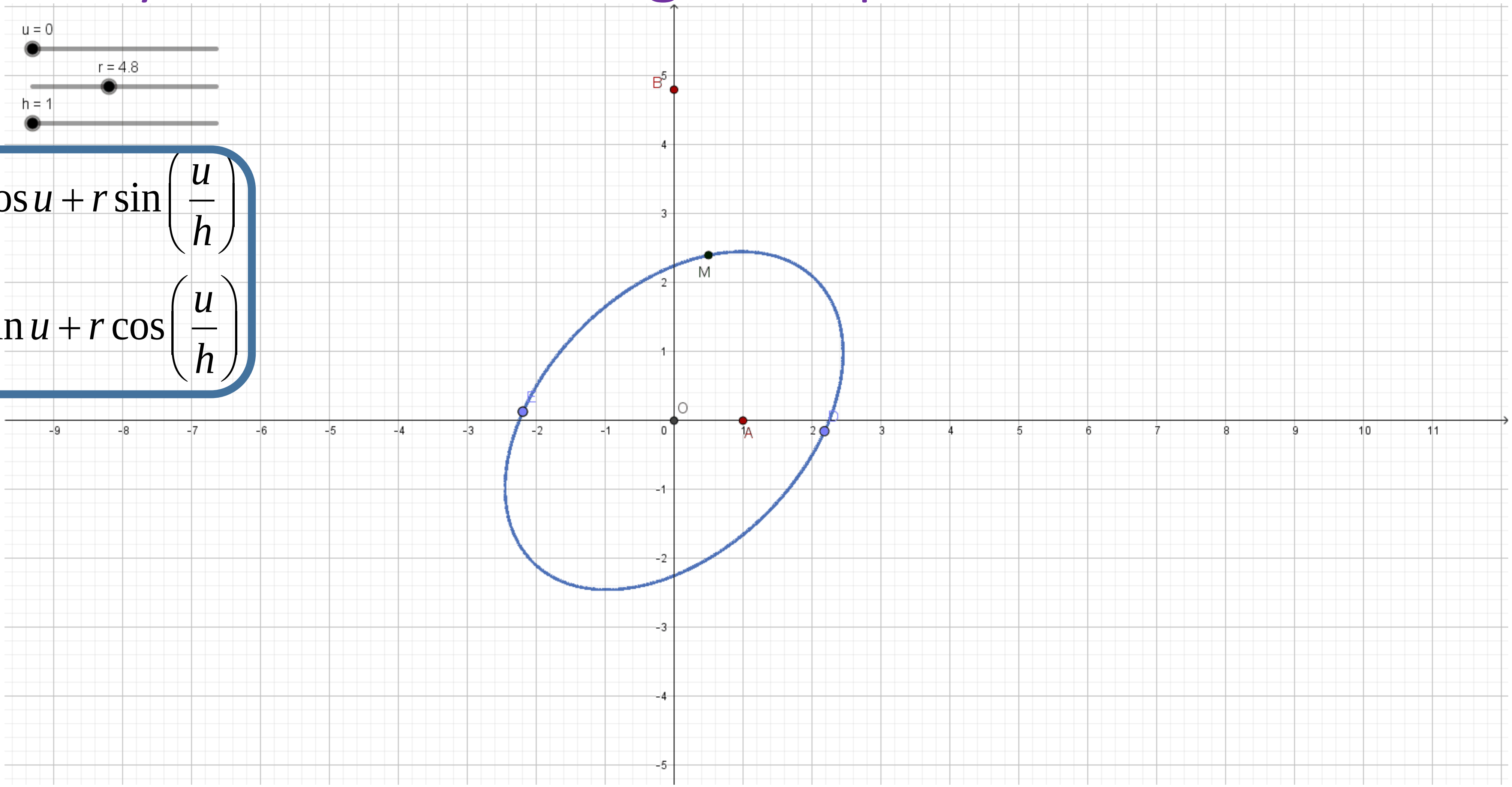
Kepler used Tycho Braha's mountains of data to find the exact direction of Mars from the earth at a whole series of times at 687.1 day intervals. If Mars can be observed from two different positions when it is at a particular point in its orbit, then one can *triangulate* the location of Mars. *Finding the direction of Mars and that of the Sun* at those times, he had a steady Mars-Sun baseline to use in constructing Mars's orbit viewed from the Earth.



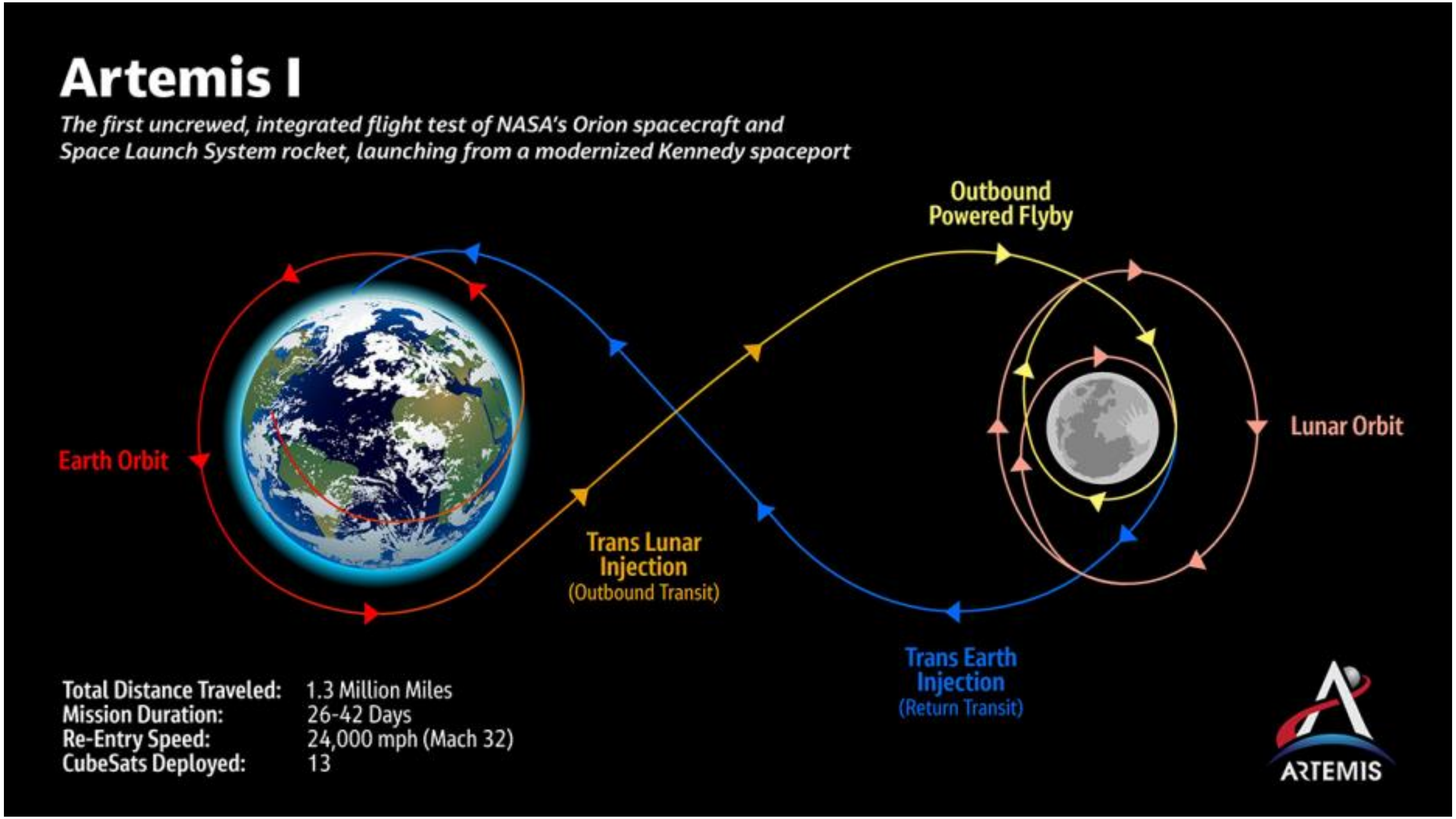
J. Kepler: Astronomia Nova (https://archive.org/stream/astronomianovaai00kepl#page/4/mode/2up)

Bicircular movements in opposite directions - modify the relative angular speeds

$$\begin{cases} x(u) = \cos u + r\sin\left(\dfrac{u}{h}\right) \\ y(u) = \sin u + r\cos\left(\dfrac{u}{h}\right) \end{cases}$$

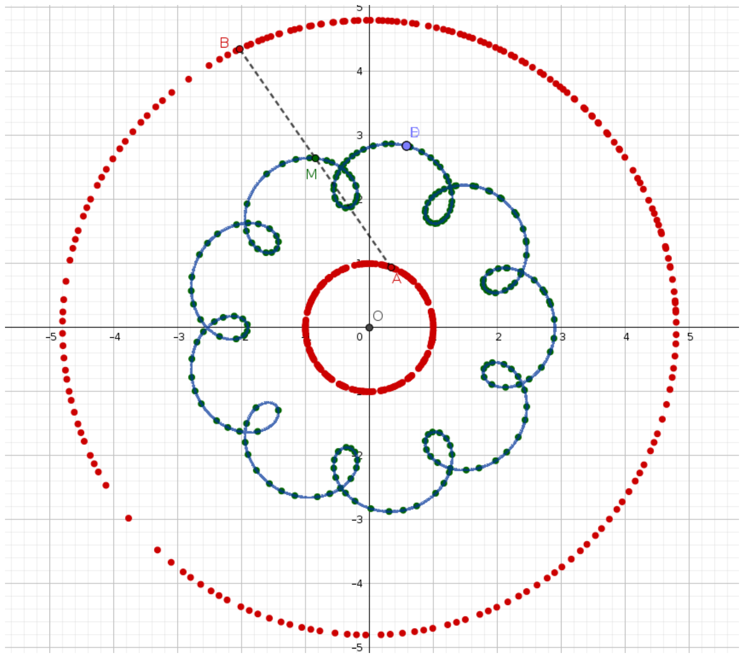


Why include retrograde movements?

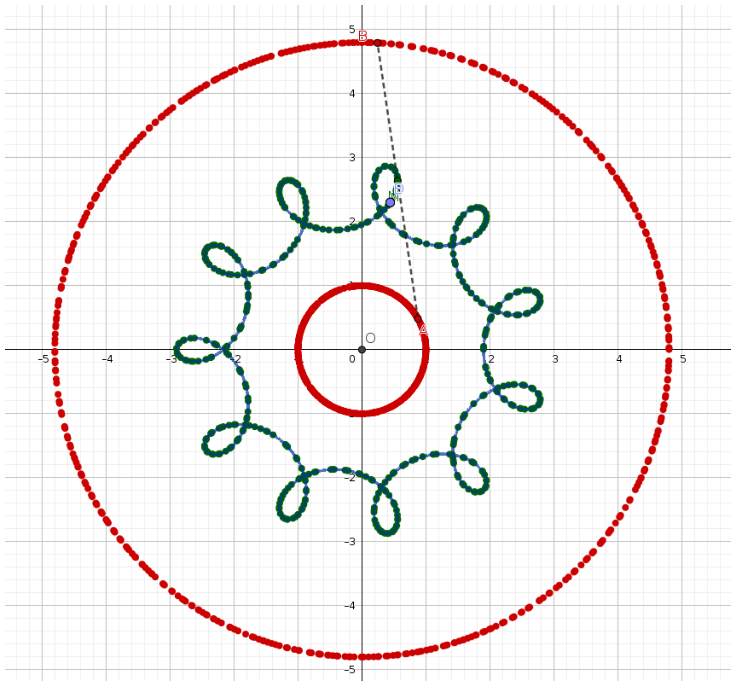


Bicircular movement

- Both in the same direction
- In reversed directions

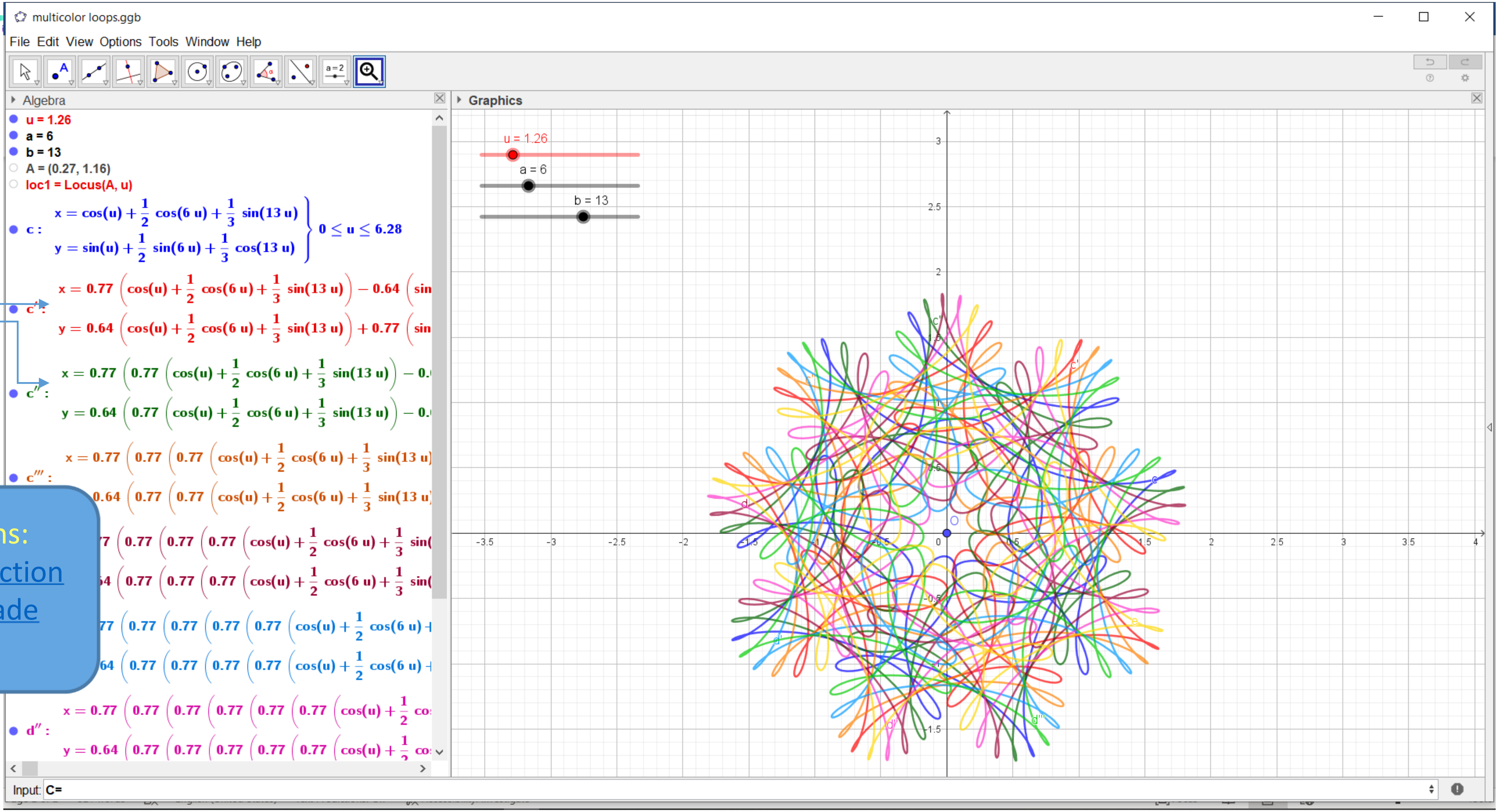


x=a cos t+r cos(bt)
y=a sin t+r sin(bt)



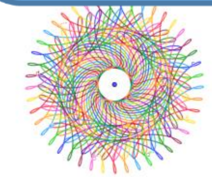
x=a cos t+r sin (bt)
y=a sin t+r cos (bt)

Tricircular movements



rotations

Animations:
1. Same direction
2. Retrograde middle



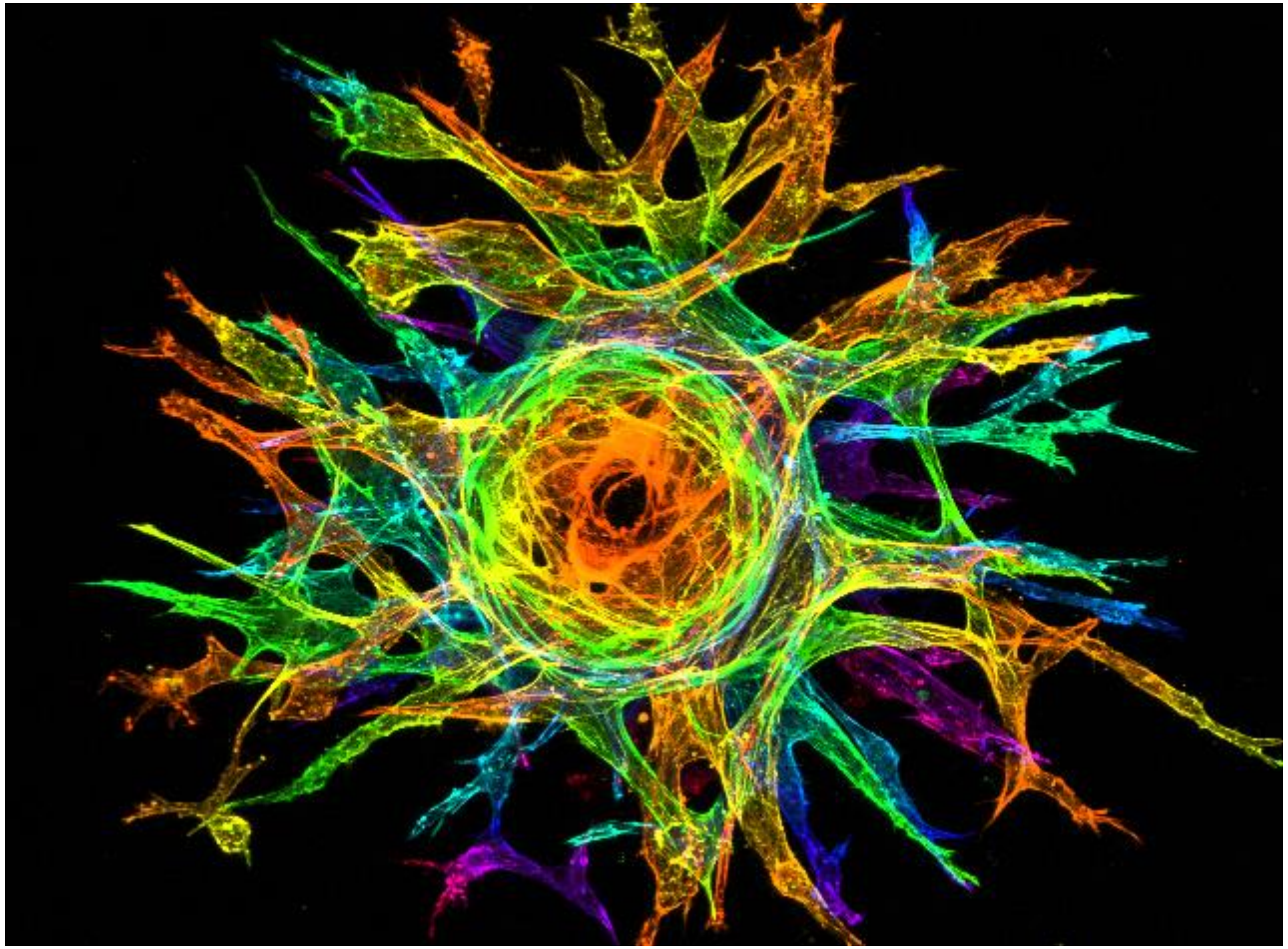
Sprouting Endothelial Cells by Karina Kinghorn of Cell Biology and Physiology.

3rd place in a Science and Art Competition 2019

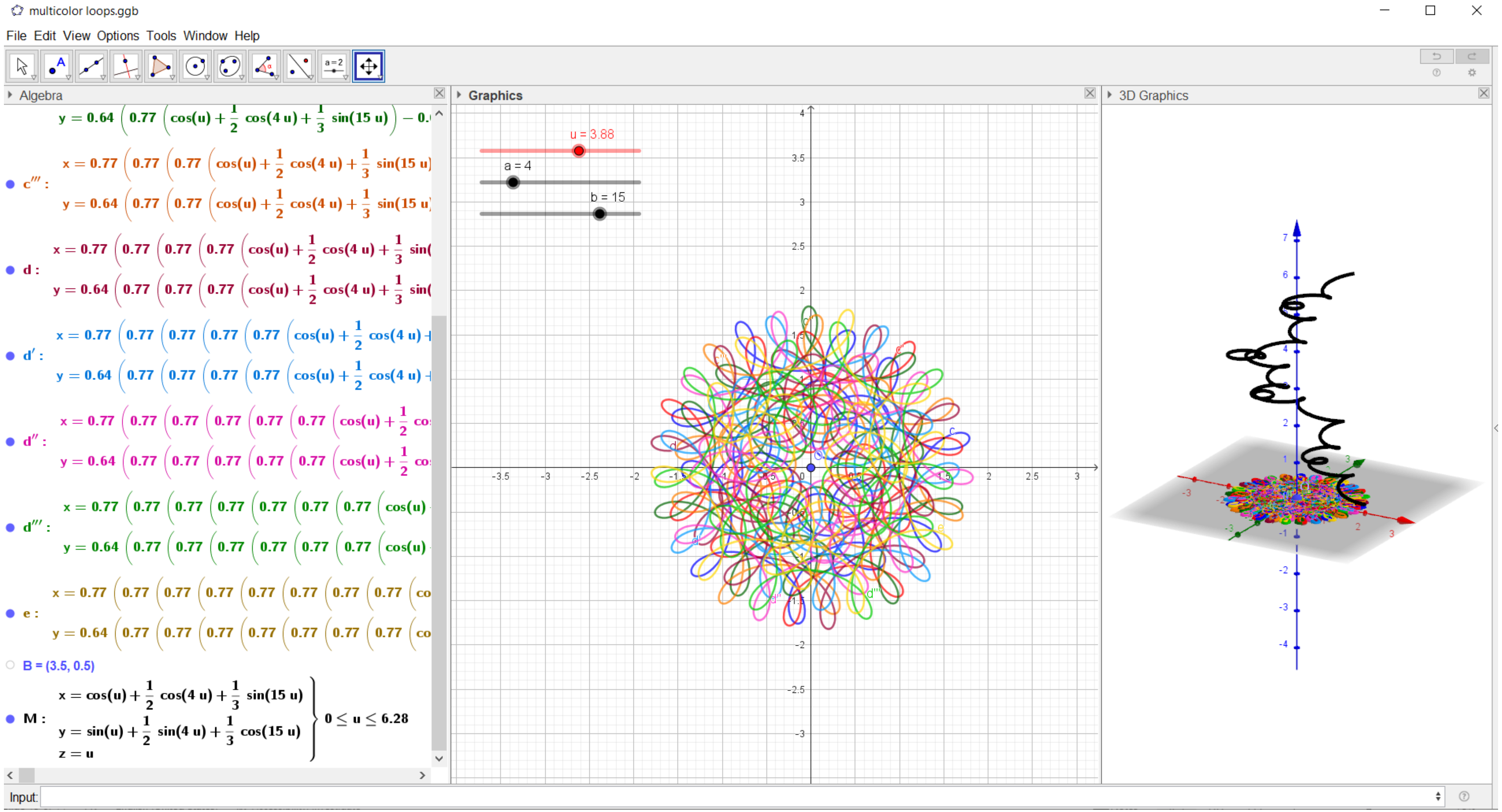
The University of North Carolina at Chapel Hill

The College for Arts and Science

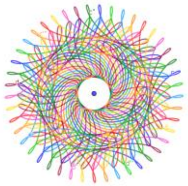
https://college.unc.edu/2020/01/science-art/



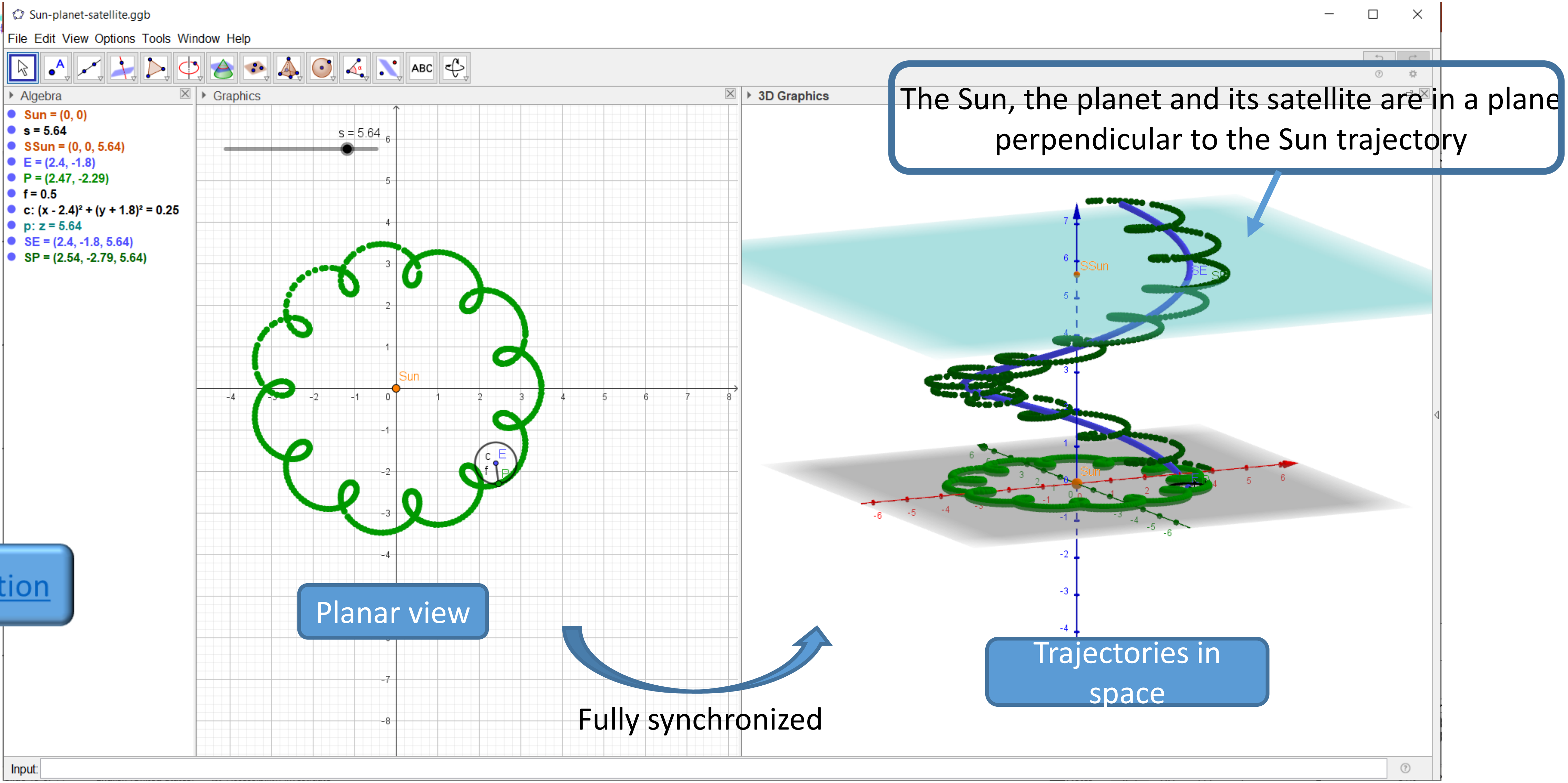
Transition to 3D



Animation



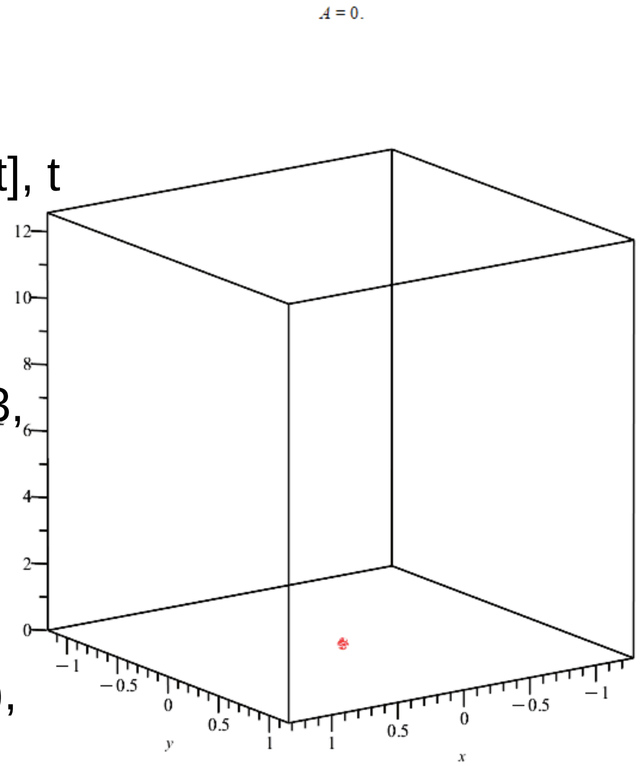
Model Sun-Planet-Satellite



Animation

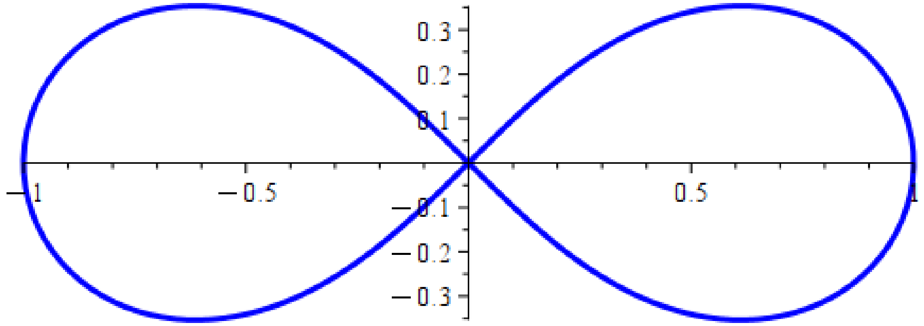
The same with Maple

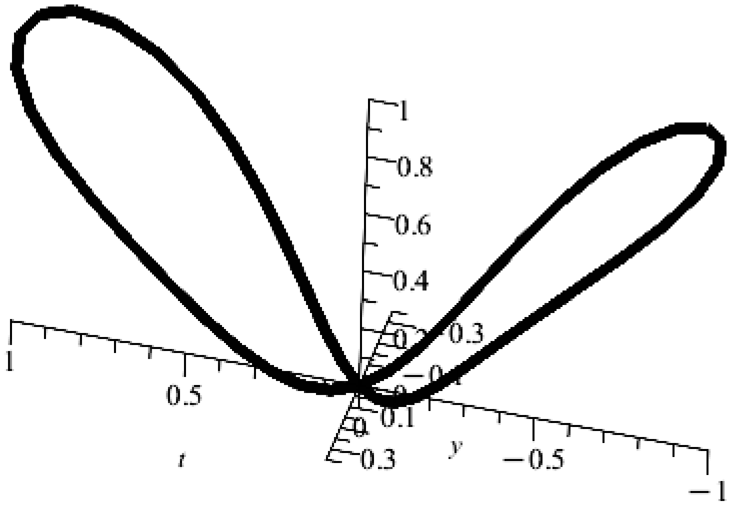
```
restart; with(plots);

c1 := spacecurve([cos(t) + 1/5*cos(12*t), sin(t) + 1/5*sin(12*t), t], t = 0 .. 4*Pi,
thickness = 3, labels = [x, y, z]);
sat := plots[animate](spacecurve, [[cos(t) + 1/5*cos(12*t), sin(t) + 1/5*sin(12*t), t], t
= 0 .. A], A = 0 .. 4*Pi, color = sienna, labels = [x, y, z]);
planet := plots[animate](spacecurve, [[cos(t), sin(t), t], t = 0 .. A], A = 0 .. 4*Pi,
thickness = 3, color = navy);
sun := plots[animate](spacecurve, [[0, 0, t], t = 0 .. A], A = 0 .. 4*Pi, thickness = 3,
color = yellow);
sunplo := plots[animate](pointplot3d, [[0, 0, A]], A = 0 .. 4*Pi, color = orange,
symbol = sphere);
display(sun, planet, sat, sunplo);
spacecraft := plots[animate](spacecurve, [[cos(t) + 1/5*cos(12*t) + 1/8*cos(14*t),
sin(t) + 1/5*sin(12*t) + 1/8*sin(14*t), t], t = 0 .. A], A = 0 .. 4*Pi, labels = [x, y, z]);
display(sunplo, sun, planet, sat, spacecraft);
```



A twisted lemniscate

$$l1 := plot\left(\left[\frac{\sin(t)}{1+\cos(t)^2}, \frac{\sin(t)\cdot\cos(t)}{1+\cos(t)^2}, t=0..2\cdot\mathrm{Pi}\right], scaling = constrained, thickness = 3, color = blue\right);$$





$$tl1 := plot3d\left(\left[\frac{\sin(t)}{1+\cos(t)^2}, \frac{\sin(t)\cdot\cos(t)}{1+\cos(t)^2}, \sin(t)^2\right], t=0..2\cdot\mathrm{Pi}, thickness = 6, axes = normal, scaling = constrained\right);$$

A twisted lemniscate – 3D printed

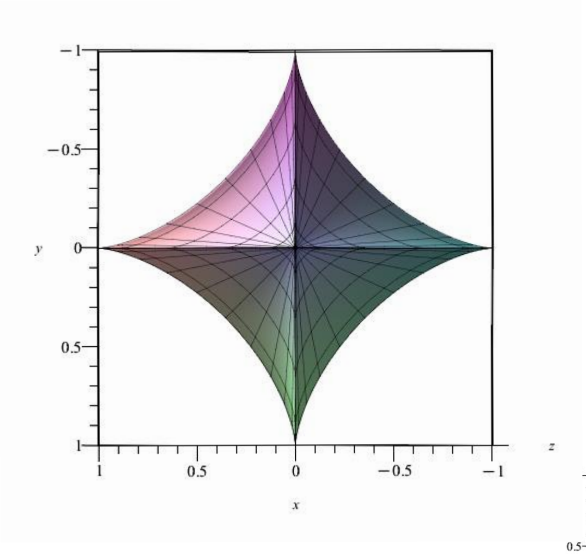
1. The equations must be identified by the software as a geometric construct
2. The curve has to be thickened in order to be 3D printed. The thickness option of the DGS/CAS is not enough
3. The entire object may not be 3D printed in one piece

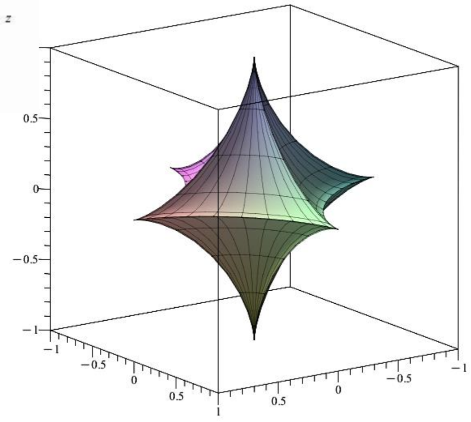
A twisted lemniscate – 3D printed

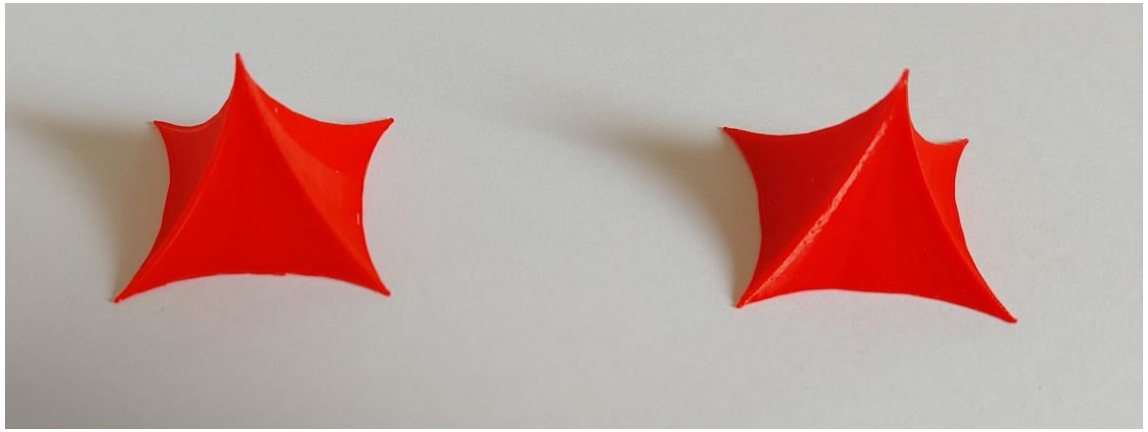
1. The equations **must be identified by the software as a geometric construct**
2. The curve has to be thickened in order to be 3D printed. The thickness option of the DGS/CAS is not enough
3. The entire object may not be 3D printed in one piece

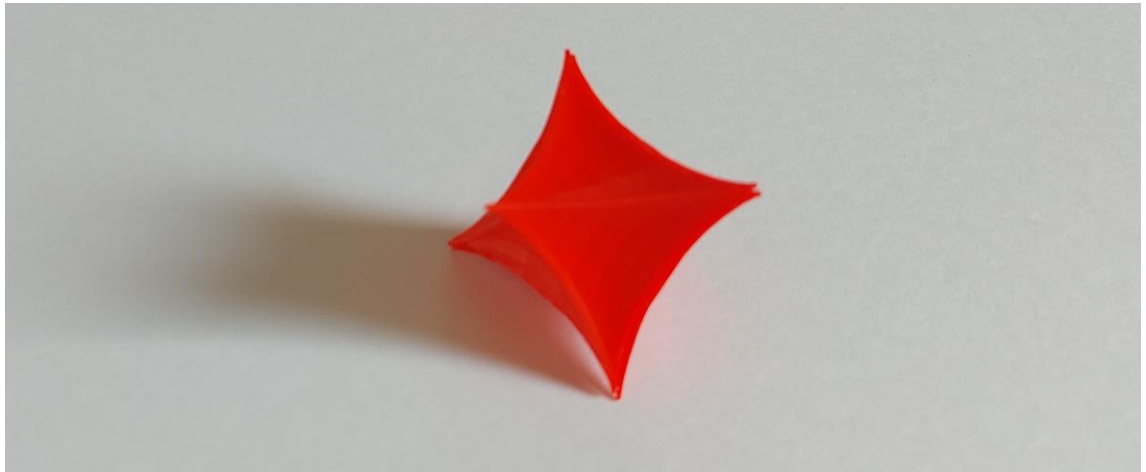
- **What is the benefit?**
- A new visualization
- Can be grasped with hands
- R. Duval (1996): not as in scientific domains as Physics, Chemistry, etc., mathematical objects cannot be grapsed and manipulated with the hands. They can be studied using *representations (numerical, symbolic, graphical, etc.)*
- **3D printing adds a new *register of representation***

An astroidal surface









plot3d([cos(u)^3*cos(v)^3, cos(u)^3*sin(v)^3, sin(u)^3], u = 0 .. 2*Pi, v = 0 .. Pi, labels = [x, y, z])

Geometric loci – general nonsense

- The computation of geometric loci is an important topic:
  - High-School level
  - undergraduate level.

← Not necessarily with the same tools

- This topic has been explored for a long time, but has a lot of novelties to offer.
- We may have used GeoGebra's Locus and/or LocusEquation commands. The output is a stills picture.
- Generally we used *animations*, both with GeoGebra and with Maple.
  - It fits more the dynamical features of the planetary orbits which were the trigger of the activities (at least at the beginning)
  - Maybe useful for students' understanding of the modeled situation
  - The same Maple programming works in 2D and in 3D
  - This reinforces man-and-machine interaction.

Some references

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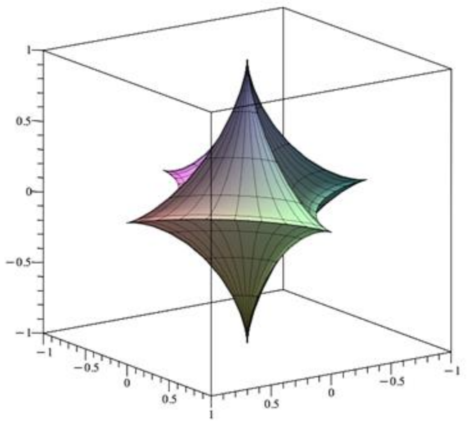

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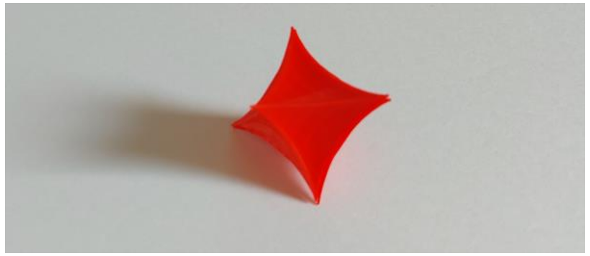

Thank you for your attention

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