

Online generation of proofs without words

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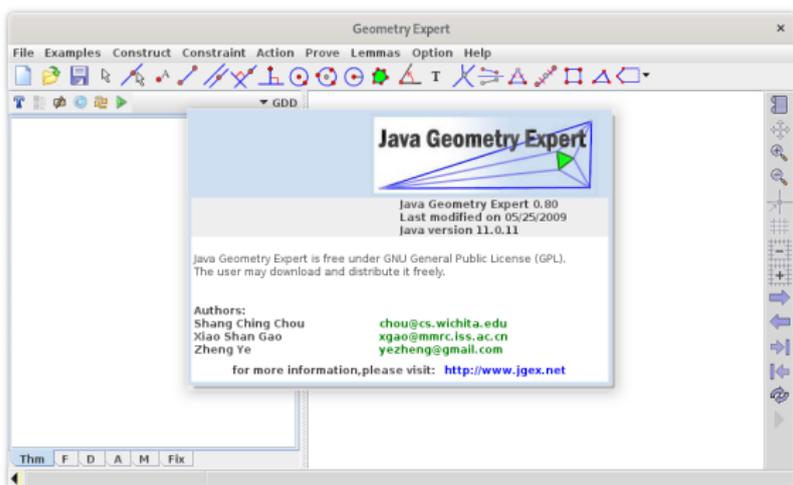
² Author is supported by the grant PID2020-113192GB-I00 from the Spanish MICINN.

We support understanding geometric relationships by presenting an overview of a work-in-progress new toolset to automatically create a **proof without words** of a construction made in GeoGebra. We combine the benefits of

- the *Geometric Deduction Database* (GDD) method (to obtain a readable proof of a geometric statement, via the codebase of the *Java Geometry Expert*) and
- the *GeoGebra* framework (to export these data as an online applet).

Java Geometry Expert (JGEX)

Available at Zheng Ye's GitHub repository

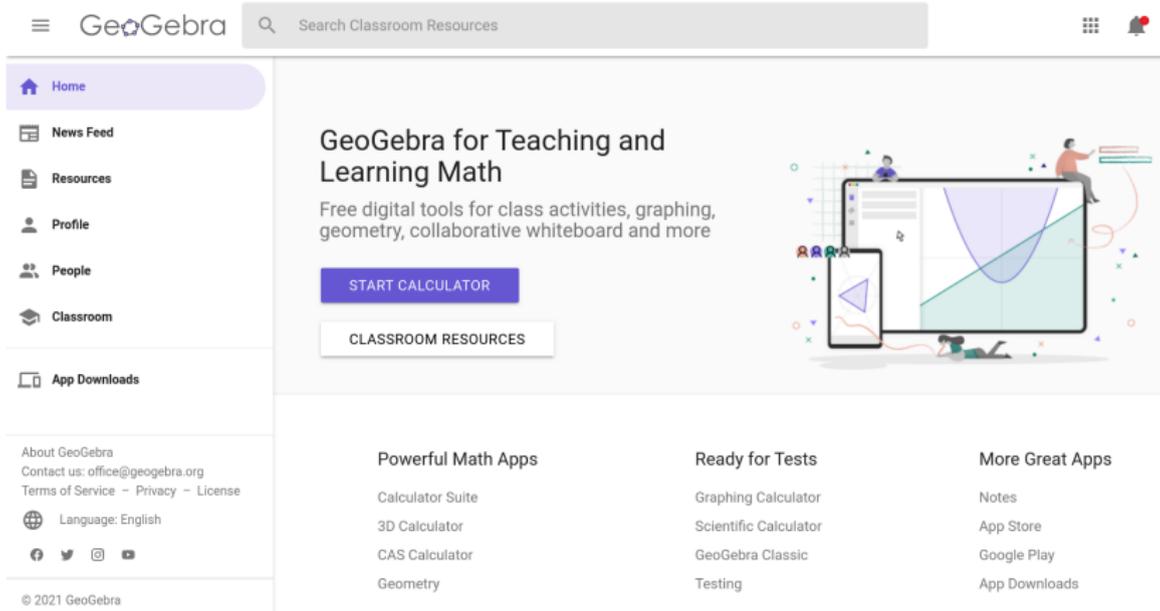


Pros:

- Written by well-known experts
- Knowledge of many decades is integrated into one tool
- Source code is freely available since December 2016 (61 forks)
- Written in Java

Cons:

- Last (minor) edit was done in July 2018
- No longer officially maintained
- Interoperability with other codebases seems non-trivial



The screenshot shows the GeoGebra website homepage. At the top, there is a navigation bar with the GeoGebra logo, a search bar for classroom resources, and a grid icon. A left sidebar contains navigation links: Home, News Feed, Resources, Profile, People, Classroom, and App Downloads. The main content area features a large banner for 'GeoGebra for Teaching and Learning Math' with a description of free digital tools and two buttons: 'START CALCULATOR' and 'CLASSROOM RESOURCES'. To the right of the banner is an illustration of people interacting with a large screen displaying a graph. Below the banner, there are three columns of featured content: 'Powerful Math Apps' (listing Calculator Suite, 3D Calculator, CAS Calculator, and Geometry), 'Ready for Tests' (listing Graphing Calculator, Scientific Calculator, GeoGebra Classic, and Testing), and 'More Great Apps' (listing Notes, App Store, Google Play, and App Downloads). The footer includes 'About GeoGebra' information, social media icons, and a copyright notice for 2021 GeoGebra.

GeoGebra

Search Classroom Resources

Home

News Feed

Resources

Profile

People

Classroom

App Downloads

About GeoGebra
Contact us: office@geogebra.org
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Language: English

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GeoGebra for Teaching and Learning Math

Free digital tools for class activities, graphing, geometry, collaborative whiteboard and more

START CALCULATOR

CLASSROOM RESOURCES

Powerful Math Apps

- Calculator Suite
- 3D Calculator
- CAS Calculator
- Geometry

Ready for Tests

- Graphing Calculator
- Scientific Calculator
- GeoGebra Classic
- Testing

More Great Apps

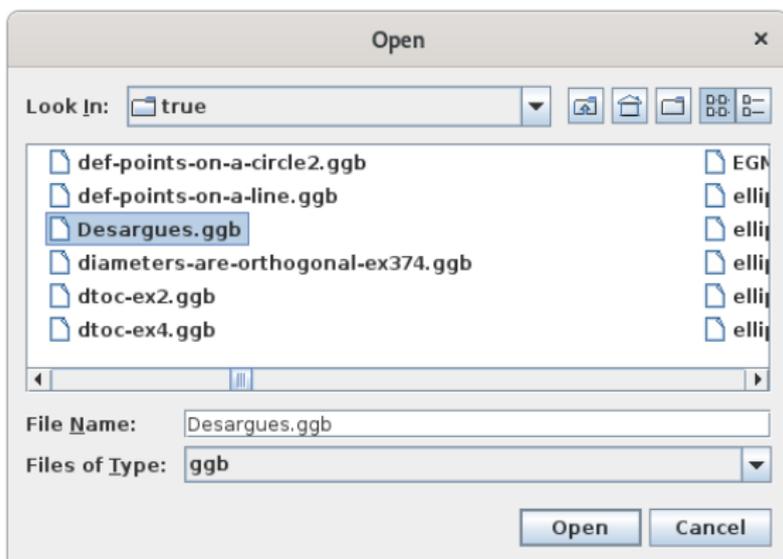
- Notes
- App Store
- Google Play
- App Downloads

Our suggestion to re-use algorithms and technology

- 1 Add GeoGebra file format import to JGEX
- 2 Add GeoGebra animated file export of a GDD proof to JGEX
- 3 Refine export animations by using feedback from the community (including teachers and students)
- 4 Copy code from JGEX to GeoGebra (including the import/export parts and the GDD database)
- 5 Connect JGEX's code to GeoGebra's Relation tool
- 6 Open the proof with words in a new window in Relation

1. Add GeoGebra file format import to JGEX

Progress: several simple examples already work



2. Add GeoGebra animated file export to JGEX

Progress: planning

The screenshot displays the Geometry Expert (GEX) software window titled "01.gex - Geometry Expert". The interface includes a menu bar (File, Examples, Construct, Constraint, Action, Prove, Lemmas, Option, Help), a toolbar with various geometric construction tools, and a main workspace showing a geometric diagram. The diagram features a triangle ABC with vertices A , B , and C . Point F is on the base BC , and point D is on the side AB . A line segment DF is drawn. Point E is on the side AC , and a line segment DE is drawn. A red line segment FG is drawn, where G is a point on DE . Right-angle symbols are shown at D and E , indicating $DF \perp DE$ and $DE \perp EC$. The left sidebar shows a list of constraints and actions:

- 1. $FG \perp DE$ (r24)
 - midp(G,DE) (by HYP)
 - 2. $FD = FE$
- 2. $FD = FE$
 - 3. $CF = DF$
 - 4. $CF = EF$
- 3. $CF = DF$ (r36)
 - midp(F,CB) (by HYP)
 - $DC \perp DB$ (by HYP)
- 4. $CF = EF$ (r36)
 - midp(F,CB) (by HYP)
 - $EC \perp EB$ (by HYP)

At the bottom of the window, there are buttons for "Thm", "F", "D", "A", "M", "Fix", and a "Move" button.

3. Refine export animations

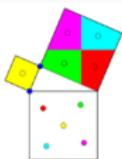
...by studying examples from GeoGebra's database (with millions of uploaded files)

Proofs Without Words

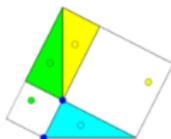
Author: [Steve Phelps](#)

Topic: [Geometry](#), [Pythagoras](#) or [Pythagorean Theorem](#)

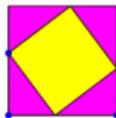
Here is a GeoGebraBook of Proofs Without Words for the Pythagorean Theorem.



Proof Without Words



Proof Without Words



Proof Without Words



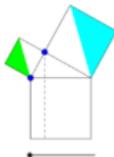
Proof Without Words



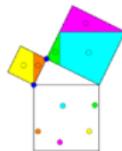
Proof Without Words



Proof Without Words



Proof Without Words



Proof Without Words

4. Copy code from JGEX to GeoGebra

The screenshot shows an IDE window titled "Java-Geometry-Expert - gdrule". The main editor displays the following text:

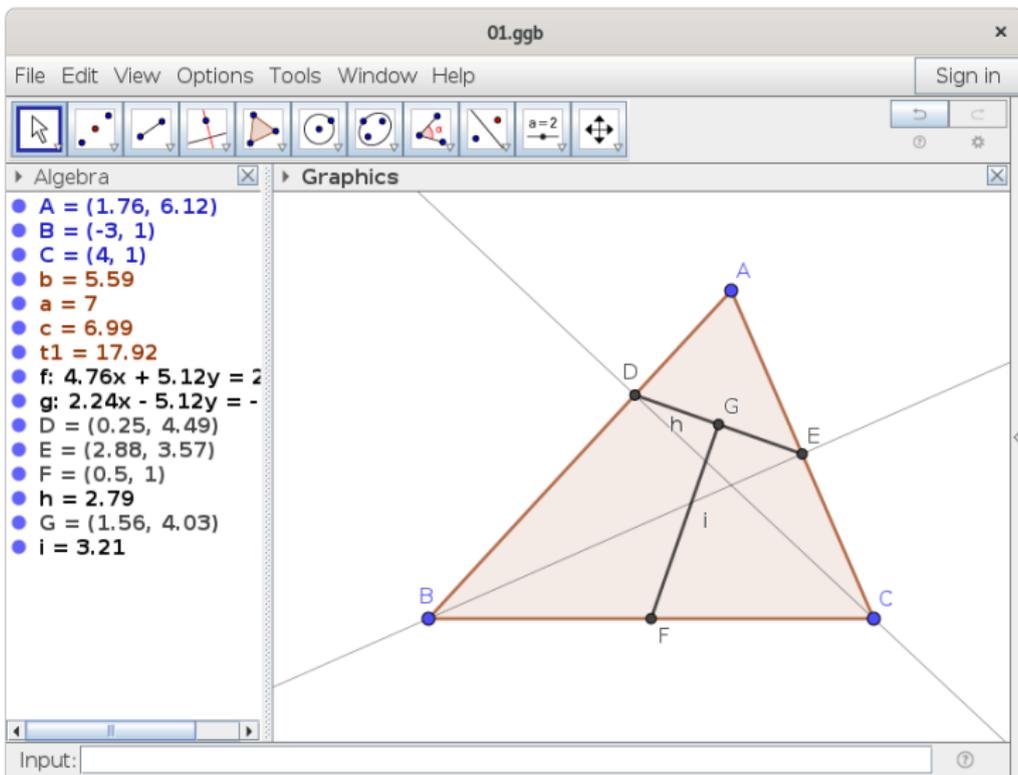
```
08 Two triangles are congruent if a pair of corresponding angles and the included side are equal.
09
10 * 27 #SAS
11 SAS(side-angle-side) congruence
12 Two triangles are congruent if a pair of corresponding sides and the included angle are equal.
13
14 * 28 #SSS
15 SSS (side-side-side) congruence
16 Two triangles are congruent if their corresponding sides are equal.
17
18 * 29 #SAS
19 SAS (side-angle-side) congruence for right triangles.
20 Two right triangle are congruent if their two corresponding cartesian edges are equal.
21
22 * 30
23 Similar triangles
24 Similar triangles are triangles that have the same shape but possibly different size.
25
26 * 31 #SAS
27 AAA (angle-angle-angle) similarity
28 If two triangles have two corresponding angles that are congruent, then the triangles are similar.
29
30 * 32 #SAS
31 SAS (side-angle-side) similarity
32 The side-angle-side (SAS) similarity test says that if two triangles have two pairs of corresponding sides in proportion and one pair of corresponding angles congruent, then the triangles are similar.
33
34 * 33 #SSS
35 SSS (side-side-side) similarity
36 If two triangles have all three pairs of sides in proportion, the triangles are similar.
37
38 * 34
39 Ratio by similarity
40
41 * 35
42 MIDDLE CONNECTION THEOREM FOR TRIANGLES
43 The line connect the midpoints of two sides of triangle is parallel to the third side and is half its length.
44
45 * 36
46 The hypotenuse of a right triangle is the diameter of the circumscribed circle of the triangle.
```

The left sidebar shows a project tree with the following structure:

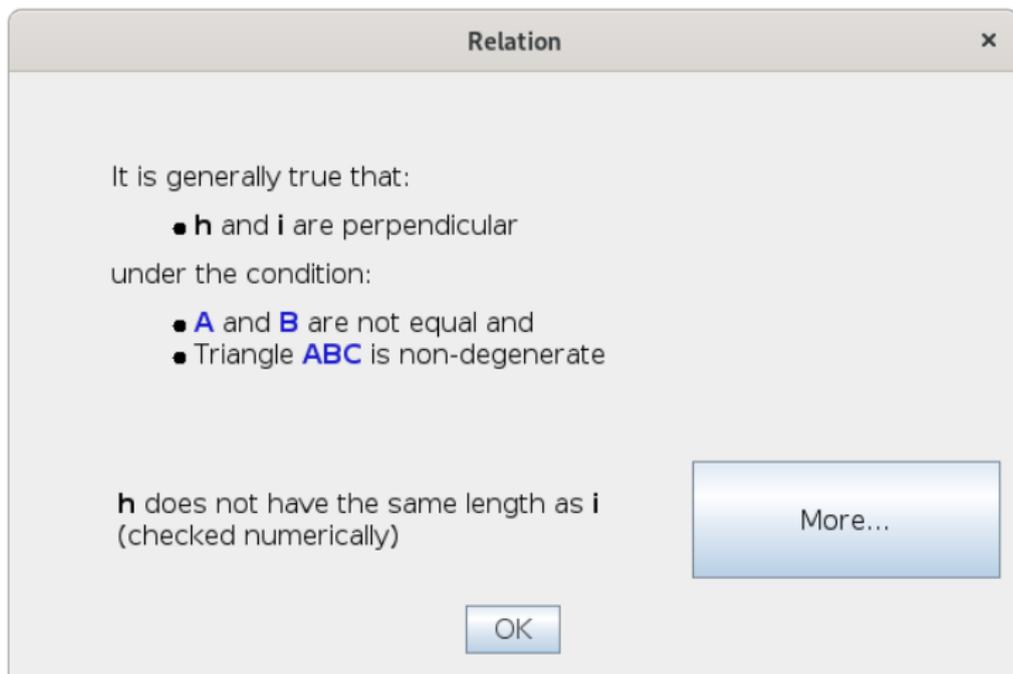
- Project
- Java-Geometry-Expert
- src
- classes
- examples
- gprover
- help
- language
- maths
- subproject
- output
- pdf
- rules
 - fulrule
 - gdrule
 - R1.gex
 - Rule1.gex
 - Rule2.gex
 - Rule3.gex
 - xxx
- UI
- wprover
 - build.xml
 - convert.jar
 - Java-Geometry-Expert.jar
 - jgex.jar
 - MANIFEST.MF
 - README.md
 - ThalesLogg
 - Untitled Diagram.xml
 - update.jar
- External Libraries
- Scratches and Consoles

The bottom status bar shows: "Ant build scripts found // Add Ant build file 'Skip' (a minute ago) 1:1 CRLF UTF-8 4 spaces jmaster"

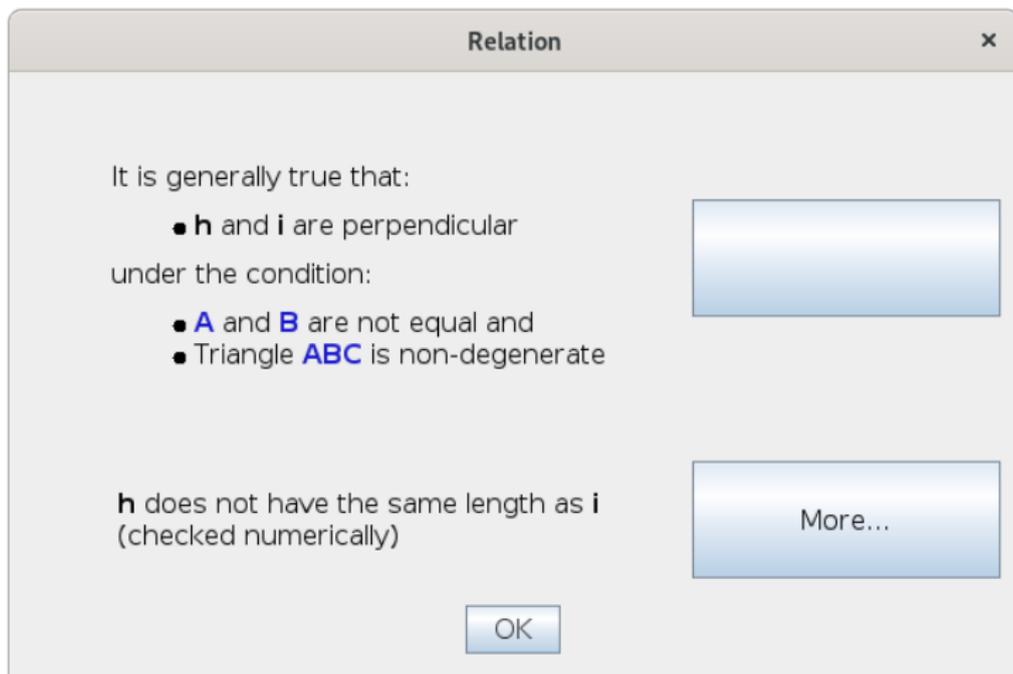
5. Connect JGEX's code to GeoGebra's Relation tool



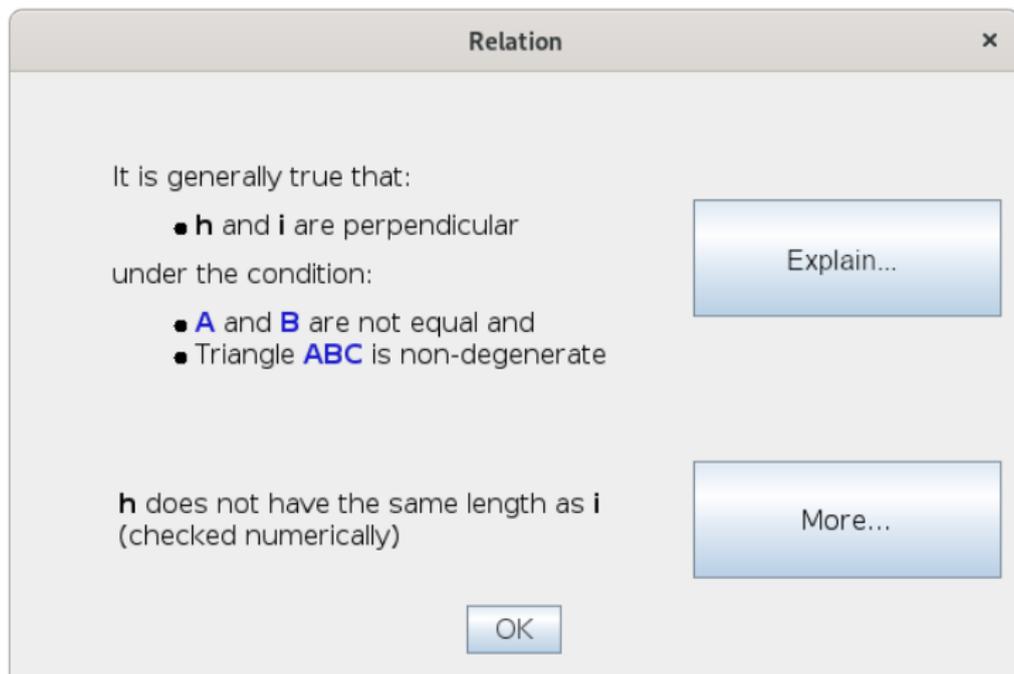
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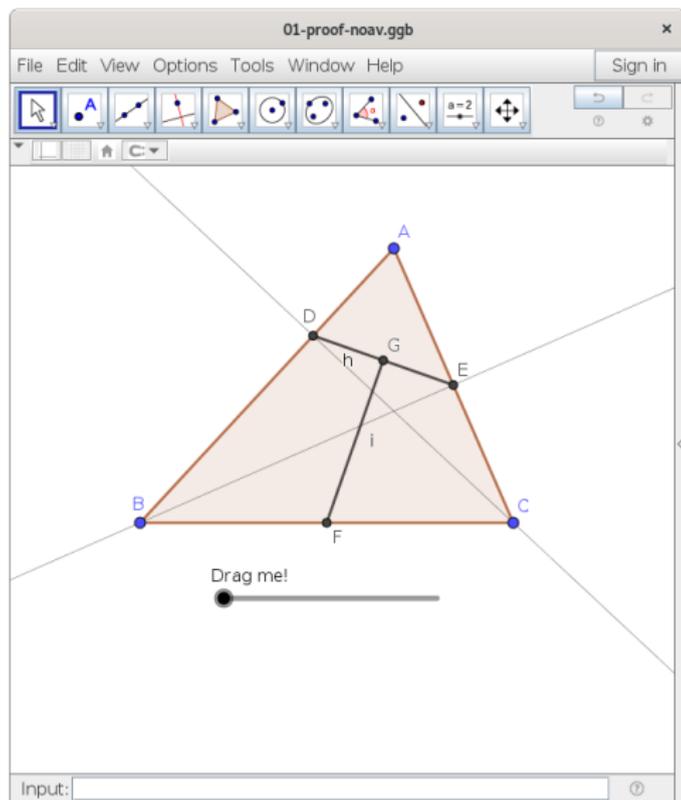
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6. Open the proof in a new window in Relation's output



6. Open the proof in a new window in Relation's output

```
mc [kovzol@kovzol-ThinkPad-E480]:~/talks/2021/adg4/01-proof-noav.ggb/uzlpj//
kovzol@kovzol-ThinkPad-E480: ~/workspace/halexus/Ja...  kovzol@kovzol-ThinkPad-E480: ~/talks/2021/adg4  mc [kovzol@kovzol-ThinkPad-E480]:~/talks/2021/adg4/...
geogebra.xml [----] 44 L:[337+14 351/547] *(13188/20271b) 0034 0x022 [*] [X]
<-----><objColor r="255" g="0" b="0" alpha="0.0"/>
<-----><layer val="0"/>
<-----><labelMode val="0"/>
<-----><coords x="-3.4907186118841054" y="-3.754722540513996" z="17.717596988050417"/>
<-----><lineStyle thickness="5" type="0" typeHidden="1" opacity="178"/>
<-----><outlyingIntersections val="false"/>
<-----><keepTypeOnTransform val="true"/>
</element>
<command name="Segment">
<-----><input a0="B" a1="F"/>
<-----><output a0="n"/>
</command>
<element type="segment" label="n">
<-----><show object="true" label="false" ev="4"/>
<-----><condition showObject="e ± 2 v e ± 5"/>
<-----><objColor r="0" g="100" b="0" alpha="0.0"/>
<-----><layer val="0"/>
<-----><labelMode val="0"/>
<-----><decoration type="2"/>
<-----><coords x="0.0" y="3.5" z="-3.5"/>
<-----><lineStyle thickness="5" type="15" typeHidden="1" opacity="178"/>
<-----><outlyingIntersections val="false"/>
<-----><keepTypeOnTransform val="true"/>
</element>
<command name="Segment">
1 SÚGÓ 2 MENT 3 JE LÖL 4 CSERE 5 MÁSOL 6 ÁTHELYEZ 7 KERES 8 TÖRÖL 9 FŐMENÜ 10 KILÉP
```

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