Towards a Structural Analysis of Interesting Geometric Theorems: Analysis of a Survey

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Extended Abstract The concept of what makes a geometric theorem "interesting" is deeply rooted in human intuition, pedagogy, and aesthetic perception. Despite its centrality in the mathematical experience, "interestingness" remains elusive to formal characterisation.

Larry Wos highlighted this profound challenge in his 31^{st} problem [8], calling for methods not only to automate theorem proving but to enable the generation and recognition of new and interesting theorems. Building upon Wos' foundational insight, Quaresma, Graziani, and Nicoletti [6] formalised a related and precise computational question: whether it is algorithmically decidable if a Turing Machine can produce interesting theorems. By reduction to Rice's theorem [7], they demonstrated that this problem is undecidable, establishing significant theoretical limits. However, this does not mean we cannot attempt to address Wos' problem (specifically its aspect concerning interestingness) in a heuristic way.

In our project, we aim to present a methodology capable of exploring the structural properties of geometric theorems that may underlie their perceived interestingness. Our approach systematically integrates human-based survey data, automated theorem proving, and Geometrographic analysis.

First, we will show how it is possible to build upon the results of a comprehensive survey conducted among mathematicians, educators, and students. Participants were asked to list geometric theorems they considered interesting and to provide qualitative explanations for their choices. This phase allowed us to capture the human dimension of mathematical interest, revealing subjective factors such as simplicity, elegance, surprise, utility, and conceptual depth.

Second, we will show how to construct formal proofs for each identified theorem using automated theorem proving tools, specifically employing the Area Method [2] (and similar methods that can be characterised geometrographically) as implemented in the GCLC prover [1,3]. These proofs will be synthetically generated, ensuring uniformity and enabling precise structural analysis.

Third, we will show how the given formal proof can be analysed using a set of quantitative metrics, known as *Geometrographic coefficients*, including:

- $\mathrm{CS}_{\mathrm{proof}}$, coefficient of simplicity giving the simplicity coefficient for the overall proof,
- $\mathrm{CS}_{\mathrm{gcl}}$, simplicity coefficient for the geometric construction (the conjecture);
- $\mathrm{CT}_{\mathrm{proof}}$, the total number of steps in the proof;