

On the Coq/Rocq Mechanization of Beeson’s “On the Notion of Equal Figures in Euclid”

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1 Introduction

When Beeson started his project [1] of computer-checking Euclid’s Elements [9] in [4] he used a set of 36 axioms. They can be split into two kinds. 20 of them are Tarski-style [12] axioms and the remaining 16 are about the notion of equal figures. Then, at the Logic Colloquium 2019, he presented his approach of defining the concept of equal figures and how to prove that it verifies the axioms needed for computer-checking Euclid’s Elements [5]. This work was completed with [2]. Our work aims at the formal verification in the Coq/Rocq proof assistant [8] of the proofs from [2]. This would not only guarantee that there is no hole in the proofs, but it would also allow to establish the mutual interpretability between the axioms used in [4] and the one used by Tarski and his co-authors in [11]. Together with the results from [7] this would also establish the mutual interpretability with Hilbert’s axioms [10]. We use the GeoCoq library [3] as a basis for this work. Up to our knowledge, there is no formal verification of the results from [2].

This work is still in progress so we opted to focus on getting more lemmas proven and submit a short extended abstract. But once the proofs completed, we will extend it to do justice to the substantial proof pearl. Here we focus on an implicitly used property in Section 7.1 of [2]. This requires stating Beeson’s definition of equal quadrilaterals used in this section. We do so in Sec. 3, after having stated the employed definitions from GeoCoq in Sec. 2. And finally the need for this implicitly used property is explained as well as the obtained results summarized in Sec. 4.