

IsaGeoCoq, a port of GeoCoq with Isabelle/HOL

Roland Coghetto

Independant scholar

« cafr-MSA2P » asbl

ADG 2025, August 1-2, Stuttgart, Germany

Two years ago... in ADG 2023

Two years ago... in ADG 2023

A teaser ... in ADG 2023

Two years ago... in ADG 2023

An Introduction to Interactive Theorem Proving in Geometry using the GeoCog library

Pierre Boutry Julien Narboux

September 2023
ADG 2023, Belgrade

◀ ◻ ▶ ◀ ◻ ▶ ◀ ≡ ▶ ◀ ≡ ▶ ≡ ↺ 🔍 ↻

Pierre Boutry, Julien Narboux (Unistra)

GeoCog tutorial

ADG 2023

1 / 26

GeoCog ported to Isabelle : IsaGeoCog

Roland Coghetto started to port GeoCoq to Isabelle. First version is available in AFP since 2021 (22klocs):



<https://www.isa-afp.org/entries/IsaGeoCq.html>

The second version is in preparation:



https://github.com/CoghettoR/IsaGeoCoq2_R1

It contains 2850 lemmas, 18 locales and 92klocs making it one of the largest Isabelle contributions (roughly 75% of GeoCog).

◀ ◻ ▶ ◀ ◻ ▶ ◀ ≡ ▶ ◀ ≡ ▶ ≡

Pierre Boutry, Julien Narboux (Unistra)

GeoCog tutorial

ADG 2023

24/26

Two years ago... in ADG 2023

An Introduction to Interactive Theorem Proving in Geometry using the GeoCoq library

Pierre Boutry Julien Narboux

September 2023
ADG 2023, Belgrade

Navigation icons

Pierre Boutry, Julien Narboux (Unistra)

GeoCoq tutorial

ADG 2023

1 / 26

GeoCoq ported to Isabelle : IsaGeoCoq

Roland Coghetto started to port GeoCoq to Isabelle. First version is available in AFP since 2021 (22klocs):



<https://www.isa-afp.org/entries/IsaGeoCoq.html>

The second version is in preparation:



https://github.com/CoghettoR/IsaGeoCoq2_R1

It contains 2850 lemmas, 18 locales and 92klocs making it one of the largest Isabelle contributions (roughly 75% of GeoCoq).

Navigation icons

Pierre Boutry, Julien Narboux (Unistra)

GeoCoq tutorial

ADG 2023

24 / 26

Plan

- Presentation of GeoCoq
- History of porting of GeoCoq
- Presentation of IsaGeoCoq
- Porting : Rock and/or Isabelle/Hol users
- Porting : non Rock and non Isabelle/Hol users
- 3 applications of IsaGeoCoq :
 - QUARESMA, Pedro et SANTOS, Vanda. *Four Geometry Problems to Introduce Automated Deduction in Secondary Schools*
 - Viviani's theorem. GEX/JGEX. *06_viviani.gex*.
 - Non-euclidean geometry (with tarski's axioms)

Presentation GeoCoq



Presentation GeoCoq



GeoCoq



GeoRocq ?

Presentation GeoCoq

Very good presentations of GeoCoq :

Presentation GeoCoq

Very good presentations of GeoCoq :

Formalisation, arithmetization and automatisation
of geometry

Julien Narboux

University of Strasbourg

ADG 2023, Belgrade



Navigation icons: back, forward, search, etc.

Julien Narboux (Unistra)

ADG 2023

1 / 63

An Introduction to Interactive Theorem Proving in
Geometry using the GeoCoq library

Pierre Boutry Julien Narboux

September 2023
ADG 2023, Belgrade

Navigation icons: back, forward, search, etc.

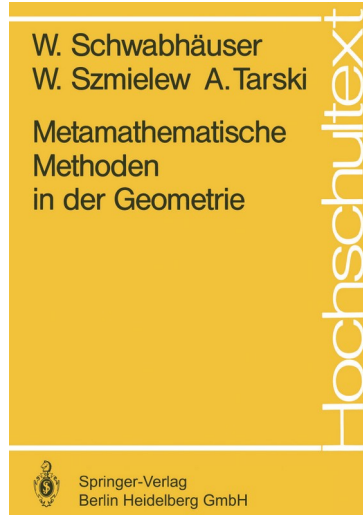
Pierre Boutry, Julien Narboux (Unistra)

GeoCoq tutorial

ADG 2023

1 / 26

Presentation GeoCoq

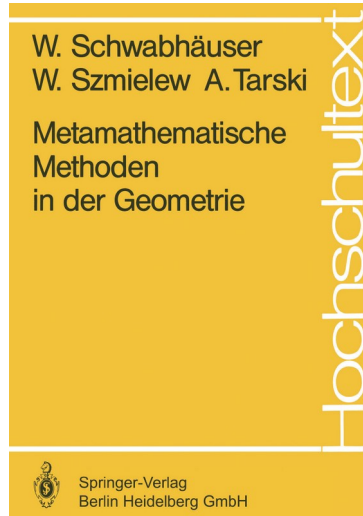


Livre SST

Teil I: Ein axiomatischer Aufbau der
euklidischen
Geometrie of the book
Metamathematische methode in des
geometrie

- 0. Einleitung
- 1. Das Tarskische Axiomensystem, kartesische Räume
- 2. Einfache Sätze über Kongruenz und Zwischenbeziehung
- 3. Konnexität der Zwischenbeziehung und Streckenvergleich
- 4. Halbgeraden und Geraden
- 5. Punktspiegelungen
- 6. Rechte Winkel
- 7. Halbebenen und Ebenen, Unterräume
- 8. Geradenspiegelungen
- 9. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume
- 10. Geradenspiegelungen
- 11. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume
- 12. Parallelität (im euklidischen Sinne)
- 13. Die Sätze von Pappus-Pascal und von Desargues
- 14. Einführung eines angeordneten Körpers
- 15. Längen von Strecken
- 16. Koordinaten

Presentation GeoCoq



Livre SST

Teil I: Ein axiomatischer Aufbau der euklidischen Geometrie of the book
Metamathematische methode in des geometrie

- 0. Einleitung
- 1. Das Tarskische Axiomensystem, kartesische Räume
- 2. Einfache Sätze über Kongruenz und Zwischenbeziehung (Ch. 02 Congruence properties)
- 3. Konnexität der Zwischenbeziehung und Streckenvergleich (Ch. 03 Between properties)
- 4. Halbgeraden und Geraden (Ch. 04 Congruence between properties - collinearity)
- 5. Punktspiegelungen (Ch. 05 Between transitivity le)
- 6. Rechte Winkel (Ch. 06 Out lines)
- 7. Halbebenen und Ebenen, Unterräume (Ch. 07 Midpoint)
- 8. Geradenspiegelungen (Ch. 08 Orthogonality)
- 9. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume (Ch 09. Planes)
- 10. Geradenspiegelungen (Ch 10. Line reflexivity 2D - line reflexivity)
- 11. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume (Ch 11. Angles)
- 12. Parallelität (im euklidischen Sinne) (Ch 12. Parallelism - parallelism using intersection decidability)
- 13. Die Sätze von Pappus-Pascal und von Desargues (Synthetic proofs of the theorems of Pappus and Desargues. Ch 13.1 - Ch 13.2 length - Ch 13.3 angles - Ch 13.4 cos - Ch 13.5 Pappus-Pascal - Ch 13.6 Desargues)
- 14. Einführung eines angeordneten Körpers (Arithmetization of geometry - Ch 14.1 Sum - Ch 14.2 Product - Ch 14.3 Ordered)
- 15. Längen von Strecken (Ch 15. Lengths)
- 16. Koordinaten (Ch 16. Coordinates - extension)

History of porting of GeoCoq

- IsaGeoCoq : not the first ;
- but to my knowledge :

History of porting of GeoCoq

- A port of the beginning of GeoCoq :

GeoCoq (Chap 2 to 9 of SST)



HOL/Light (by John Harrison)



with miz3

Mizar (by William Richter, Adam Grabowski and Jesse Alama)
2014 : GTARSKI1.MIZ (in MML)

- GTARSKI3.MIZ (2017), ... GTARSKI5.MIZ (2023) not porting but translation of chap 2 to 9 of SST.
(by Adam Grabowski and Roland Coghetto – Mizar – (in MML))

History of porting of GeoCoq

- A port of the latest results from GeoCoq :

GeoCoq (Euclid Book 1)



Dedukti (by Yoan Geran)

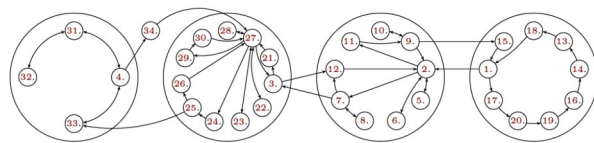


Interactive proof assistants
(the family of simple types and dependent types : Isabelle/HOL,...)

Presentation IsaGeoCoq

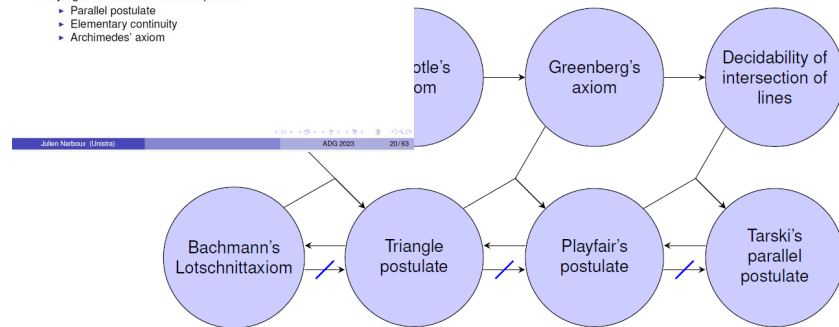
First project

Sorting 34 postulates

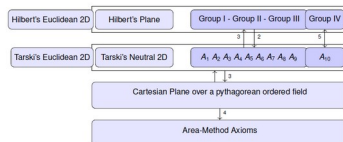


12

- Joint work with Charly Gries and Gabriel Braun
- Mechanizing proofs of Euclid's **statements**
- Not Euclid's proofs!
- Trying to minimize the assumptions:
 - Parallel postulate
 - Elementary continuity
 - Archimedes' axiom



Overview of the axiom systems



²Gabriel Braun, Pierre Boutry, and Julien Narboux (June 2016). "From Hilbert to Tarski". In: Eleventh International Workshop on Automated Deduction in Geometry. Proceedings of ADG 2016

³Gabriel Braun and Julien Narboux (Sept. 2012). "From Tarski to Hilbert". English. In: Post-proceedings of Automated Deduction in Geometry 2012. Vol. 7993. LNCS

⁴Pierre Boutry, Gabriel Braun, and Julien Narboux (2019). "Formalization of the Arithmetization of Euclidean Plane Geometry and Applications". In: Journal of Symbolic Computation 98

⁵Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: Journal of Automated Reasoning

and continuity axioms: a
In:

ADG 2023 40 / 63

Second project

- Joint work with Michael Beeson and Freek Wiedijk ⁶
- Formalizing Euclid's **proofs**
- A not minimal axiom system
- Filling the gaps in Euclid

⁶Michael Beeson, Julien Narboux, and Freek Wiedijk (2019). "Proof-checking Euclid". In: Annals of Mathematics and Artificial Intelligence 85,2-4

Julien Narboux (Linares) ADG 2023 21 / 63

a) ADG 2023 39 / 63

Presentation IsaGeoCoq



Dependant type theory

Simple type theory (HOL)

- IsaGeoCoq, initial version in AFP, 2021
- IsaGeoCoq2_R1, working version, (in WIP in github but now archived in GITWEB of association). Obsolète.
- IsaGeoCoq, refactoring of IsaGeoCoq2_R1, in dev-AFP, 2025

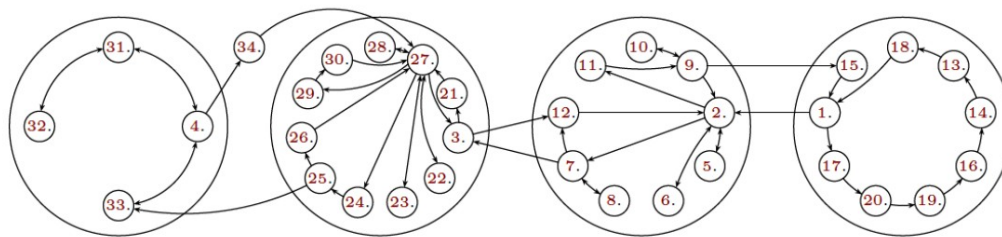
Presentation IsaGeoCoq

- IsaGeoCoq, AFP 2021
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.
 - 0. Einleitung
 - 1. Das Tarskische Axiomensystem, kartesische Räume
 - 2. Einfache Sätze über Kongruenz und Zwischenbeziehung (Ch. 02 Congruence properties)
 - 3. Konnexität der Zwischenbeziehung und Streckenvergleich (Ch. 03 Between properties)
 - 4. Halbgeraden und Geraden (Ch. 04 Congruence between properties - collinearity)
 - 5. Punktspiegelungen (Ch. 05 Between transitivity le)
 - 6. Rechte Winkel (Ch. 06 Out lines)
 - 7. Halbebenen und Ebenen, Unterräume (Ch. 07 Midpoint)
 - 8. Geradenspiegelungen (Ch. 08 Orthogonality)
 - 9. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume (Ch 09. Planes)
 - 10. Geradenspiegelungen (Ch 10. Line reflexivity 2D - line reflexivity)
 - 11. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume (Ch 11. Angles)
 - 12. Parallelität (im euklidischen Sinne) (Ch 12. Parallelism - parallelism using intersection decidability)
 - 13. Die Sätze von Pappus-Pascal und von Desargues (Synthetic proofs of the theorems of Pappus and Desargues. Ch 13.1 - Ch 13.2 length - Ch 13.3 angles—Ch 13.4 cos—Ch 13.5 Pappus-Pascal—Ch 13.6 Desargues)
 - 14. Einführung eines angeordneten Körpers (Arithmetization of geometry—Ch 14.1 Sum—Ch 14.2 Product—Ch 14.3 Ordered)
 - 15. Längen von Strecken (Ch 15. Lengths)
 - 16. Koordinaten (Ch 16. Coordinates—extension)

Presentation IsaGeoCoq

- IsaGeoCoq, AFP 2021
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Sorting 34 postulates



12

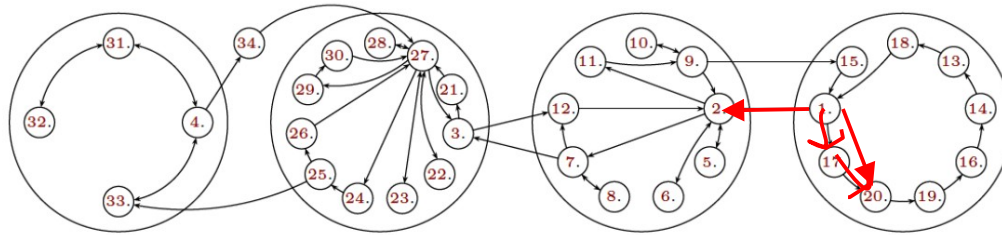
¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Navigation icons: back, forward, search, etc.

Presentation IsaGeoCoq

- IsaGeoCoq, AFP 2021
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Sorting 34 postulates



12

¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Navigation icons: back, forward, search, etc.

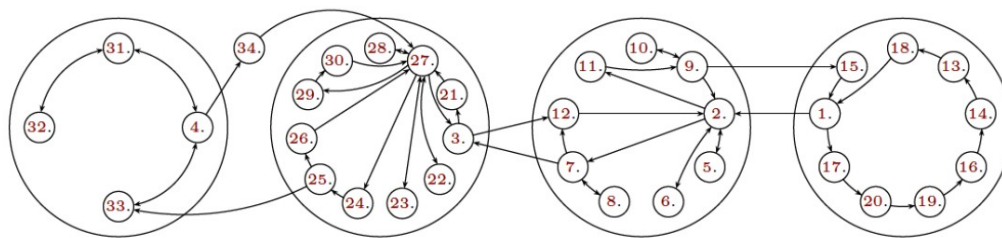
Presentation IsaGeoCoq

- IsaGeoCoq, dev AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.
 - 0. Einleitung
 - 1. Das Tarskische Axiomensystem, kartesische Räume
 - 2. Einfache Sätze über Kongruenz und Zwischenbeziehung (Ch. 02 Congruence properties)
 - 3. Konnexität der Zwischenbeziehung und Streckenvergleich (Ch. 03 Between properties)
 - 4. Halbgeraden und Geraden (Ch. 04 Congruence between properties - collinearity)
 - 5. Punktspiegelungen (Ch. 05 Between transitivity le)
 - 6. Rechte Winkel (Ch. 06 Out lines)
 - 7. Halbebenen und Ebenen, Unterräume (Ch. 07 Midpoint)
 - 8. Geradenspiegelungen (Ch. 08 Orthogonality)
 - 9. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume (Ch 09. Planes)
 - 10. Geradenspiegelungen (Ch 10. Line reflexivity 2D - line reflexivity)
 - 11. Kongruenz und Größenvergleich von Winkeln, Kongruenzsätze, Orthogonalität für Unterräume (Ch 11. Angles)
 - 12. Parallelität (im euklidischen Sinne) (Ch 12. Parallelism - parallelism using intersection decidability)
 - 13. Die Sätze von Pappus-Pascal und von Desargues (Synthetic proofs of the theorems of Pappus and Desargues. Ch 13.1 - Ch 13.2 length - Ch 13.3 angles - Ch 13.4 cos - Ch 13.5 Pappus-Pascal - Ch 13.6 Desargues)
 - 14. Einführung eines angeordneten Körpers (Arithmetization of geometry - Ch 14.1 Sum - Ch 14.2 Product - Ch 14.3 Ordered)
 - 15. Längen von Strecken (Ch 15. Lengths)
 - 16. Koordinaten (~~Ch 16. Coordinates - extension~~)

Presentation IsaGeoCoq

- IsaGeoCoq, dev
AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Sorting 34 postulates



12

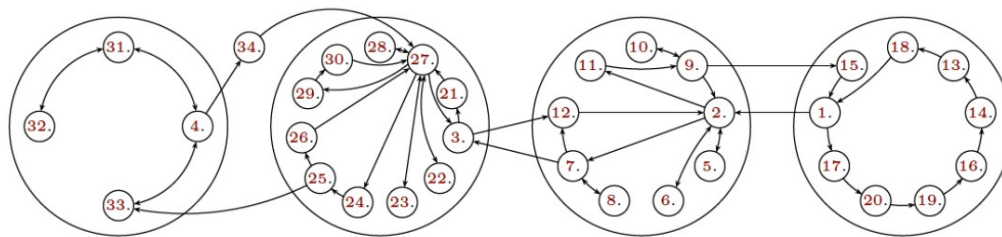
¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Navigation icons: back, forward, search, etc.

Presentation IsaGeoCoq

- IsaGeoCoq, dev
AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Sorting 34 postulates



12

All arrows are proven

¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Presentation IsaGeoCog

- IsaGeoCoq, dev AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Sorting 34 postulates

(*
Pierre Boutry, Charly Gries, Julien Narboux, Pascal Schreck. Parallel postulates and continuity
axioms: a mechanized study in intuitionistic logic using Coq. Journal of Automated Reasoning,
Springer Verlag, 2019, 62 (1), pp.68. [ff10.1007/s10817-017-9422-8](https://doi.org/10.1007/s10817-017-9422-8)ff. [ffhal-01178236v2](https://doi.org/10.1007/s10817-017-9422-8)f

Theorem 10 In Archimedean neutral geometry, Postulates 1-34 are equivalent

*)

theorem Thm10:

```
assumes "archimedes axiom"
```

```
shows "(Postulate01  $\longleftrightarrow$  Postulate02)  $\wedge$ 
```

$$(\text{Postulate01} \longleftrightarrow \text{Postulate03}) \wedge$$
$$(\text{Postulate01} \longleftrightarrow \text{Postulate04}) \wedge$$
$$(\text{Postulate01} \longleftrightarrow \text{Postulate05}) \wedge$$
$$(\text{Postulate01} \iff \text{Postulate06}) \wedge$$
$$(\text{Postulate01} \longleftrightarrow \text{Postulate07}) \wedge$$
$$(\text{Postulate01} \iff \text{Postulate08}) \wedge$$
$$(\text{Postulate01} \iff \text{Postulate09}) \wedge$$

¹²Pierre Boutry et al. (2017). “Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq”. In:

Journal of Automated Reasoning

Presentation IsaGeoCoq

- IsaGeoCoq, dev
AFP 2025
- Title : Tarski's Parallel
Postulate implies the 5th
Postulate of Euclid, the
Postulate of Playfair and
the original Parallel
Postulate of Euclid.

Sorting 34 postulates

□ Tarski_Postulate_Parallels.thy (~/IsaGeoCoq2_R1-f848c1d/)

```
(Postulate01  $\longleftrightarrow$  Postulate32)  $\wedge$ 
(Postulate01  $\longleftrightarrow$  Postulate33)  $\wedge$ 
(Postulate01  $\longleftrightarrow$  Postulate34)  $\wedge$ 
(Postulate01  $\longleftrightarrow$  Postulate35)"
using InterAx5 InterCycle3 Postulate01_def Postulate03_def playfair__alternate_interior
tarski_s_euclid_implies_playfair_s_postulate alternate_interior__triangle
Cycle_2 Postulate03_def Postulate20_def assms legendre_s_third_theorem InterAx5
Cycle_3 P04_P33 P25_33 P34_P27 P4_P34 assms InterCycle1bis Postulate02_def
Postulate12_def playfair_bis__playfair Postulate07_def Postulate08_def
alternate_interior__consecutive_interior consecutive_interior__alternate_interior
equivalent_postulates_without_decidability_of_intersection_of_lines
equivalent_postulates_without_decidability_of_intersection_of_lines
InterAx5 InterCycle1bis InterCycle3 Postulate02_def Postulate03_def
Postulate12_def playfair__alternate_interior alternate_interior__triangle
playfair_bis__playfair equivalent_postulates_without_decidability_of_intersection_of_lines
Cycle_2 Cycle_1 Postulate11_def Postulate15_def
inter_dec_plus_par_perp_perp_imply_triangle_circumscription
universal_posidonius_postulate_perpendicular_transversal_postulate
Postulate03_def Postulate21_def legendre_s_second_theorem
triangle__existential_triangle assms Thm10_1 Cycle_3 assms Thm10_1 P31_P04
P31_P04 P31_P32 P04_P33 P4_P34 P34_P27 Thm10_3 P01_P35 P35_27 by blast (* 3 sec *)
```

Julien Narboux (Unistra)

ADG 2023

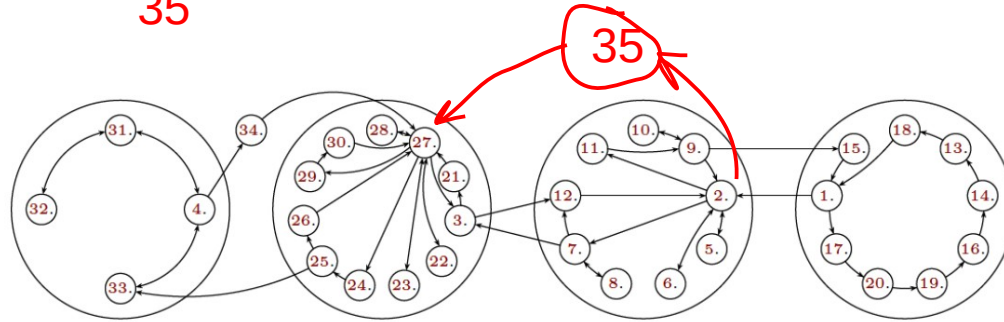
40 / 63

Presentation IsaGeoCoq

- IsaGeoCoq, dev AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Sorting ~~34~~ postulates

35



12

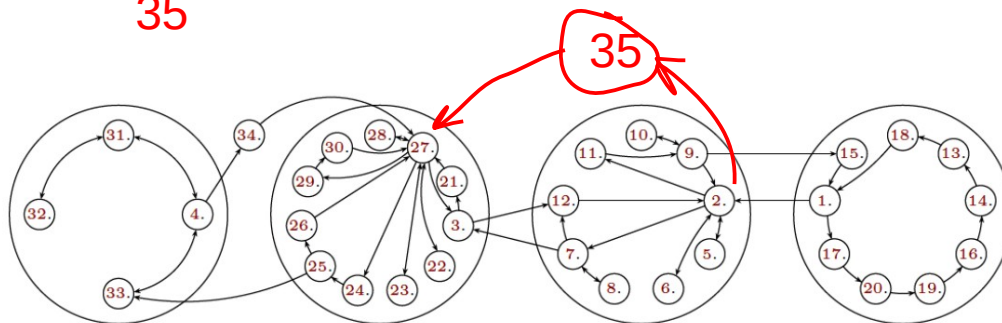
¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Presentation IsaGeoCoq

- IsaGeoCoq, dev
AFP 2025
- Title : Tarski's Parallel
Postulate implies the 5th
Postulate of Euclid, the
Postulate of Playfair and
the original Parallel
Postulate of Euclid.

Sorting ~~34~~ postulates

35



12

$35 \rightarrow 27$: GeoCoq, IsaGeoCoq : existential_playfair__rah
 $2 \rightarrow 35$: GeoCoq, IsaGeoCoq : playfair__existential_playfair

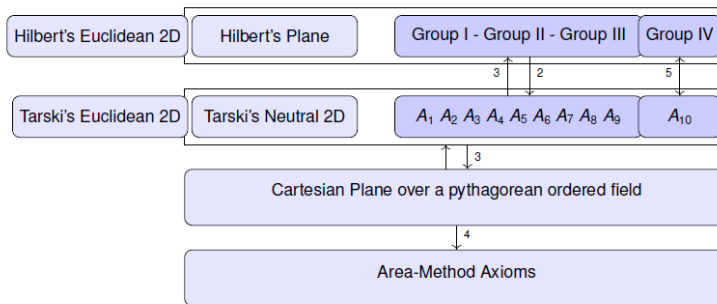
¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Navigation icons: back, forward, search, etc.

Presentation IsaGeoCoq

- IsaGeoCoq, dev AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Overview of the axiom systems



²Gabriel Braun, Pierre Boutry, and Julien Narboux (June 2016). "From Hilbert to Tarski". In: [Eleventh International Workshop on Automated Deduction in Geometry](#). Proceedings of ADG 2016

³Gabriel Braun and Julien Narboux (Sept. 2012). "From Tarski to Hilbert". English. In: [Post-proceedings of Automated Deduction in Geometry 2012](#). Vol. 7993. LNCS

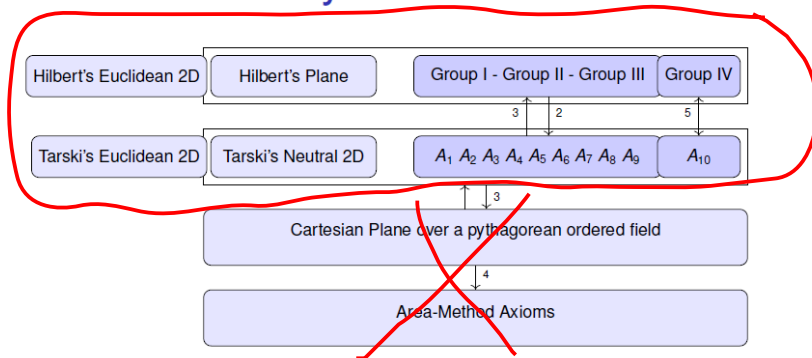
⁴Pierre Boutry, Gabriel Braun, and Julien Narboux (2019). "Formalization of the Arithmetization of Euclidean Plane Geometry and Applications". In: [Journal of Symbolic Computation](#) 98

⁵Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Presentation IsaGeoCoq

- IsaGeoCoq, dev AFP 2025
- Title : Tarski's Parallel Postulate implies the 5th Postulate of Euclid, the Postulate of Playfair and the original Parallel Postulate of Euclid.

Overview of the axiom systems



²Gabriel Braun, Pierre Boutry, and Julien Narboux (June 2016). "From Hilbert to Tarski". In: [Eleventh International Workshop on Automated Deduction in Geometry. Proceedings of ADG 2016](#)

³Gabriel Braun and Julien Narboux (Sept. 2012). "From Tarski to Hilbert". English. In: [Post-proceedings of Automated Deduction in Geometry 2012. Vol. 7993. LNCS](#)

⁴Pierre Boutry, Gabriel Braun, and Julien Narboux (2019). "Formalization of the Arithmetization of Euclidean Plane Geometry and Applications". In: [Journal of Symbolic Computation 98](#)

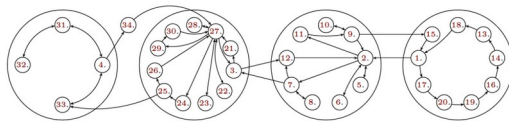
⁵Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: [Journal of Automated Reasoning](#)

Presentation IsaGeoCoq

- IsaGeoCoq, dev AFP 2025

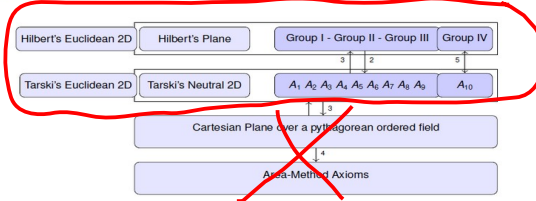
- Title : Tarski's Parallel Postulate implies the Parallel Postulate of Euclid, the Parallel Postulate of Playfair and the original Parallel Postulate

Sorting 34 postulates



12

Overview of the axiom systems



²Gabriel Braun, Pierre Boutry, and Julien Narboux (June 2016). "From Hilbert to Tarski". In: Eleventh International Workshop on Automated Deduction in Geometry. Proceedings of ADG 2016

³Gabriel Braun and Julien Narboux (Sept. 2012). "From Tarski to Hilbert". English. In: Post-proceedings of Automated Deduction in Geometry 2012. Vol. 7993. LNCS

⁴Pierre Boutry, Gabriel Braun, and Julien Narboux (2019). "Formalization of the Arithmetization of Euclidean Plane Geometry and Applications". In: Journal of Symbolic Computation 98

⁵Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In: Journal of Automated Reasoning

Angles)

- 12. Parallelität (im euklidischen Sinne) (Ch 12. Parallelism - parallelism using intersection decidability)
- 13. Die Sätze von Pappus-Pascal und von Desargues (Synthetic proofs of the theorems of Pappus and Desargues. Ch 13.1 - Ch 13.2 length - Ch 13.3 angles - Ch 13.4 cos - Ch 13.5 Pappus-Pascal - Ch 13.6 Desargues)
- 14. Einführung eines angeordneten Körpers (Arithmetization of geometry - Ch 14.1 Sum - Ch 14.2 Product - Ch 14.3 Ordered)
- 15. Längen von Strecken (Ch 15. Lengths)
- 16. Koordinaten (Ch 16. Coordinates - extension)

ung (Ch. 02 Congruence properties)
gleich (Ch. 03 Between properties)
neen properties - collinearity)

int)

uenzsätze, Orthogonalität für Unterräume (Ch 09.

ne reflexivity)

gruenzsätze, Orthogonalität für Unterräume (Ch 11.

¹²Pierre Boutry et al. (2017). "Parallel postulates and continuity axioms: a mechanized study in intuitionistic logic using Coq". In:

Journal of Automated Reasoning

Presentation IsaGeoCoq

- IsaGeoCoq, dev
AFP 2025
- Title : Tarski's Parallel
Postulate implies the 5th
Postulate of Euclid, the
Postulate of Playfair and
the original Parallel
Postulate of Euclid.

BONUS : port of highschool exemples of
GeoCoq : Varignon, Euler line, sesamath,...

Porting

- Rock and/or Isabelle/HOL users
- For non-Rock and non-Isabelle/Hol users

Porting : Rock and/or Isabelle/HOL users

- Very technical aspects. I am not an expert not with Rock, not with Isabelle/HOL.
- Manual, reading line by line GeoCoq code, with CoqIde
- Importance of reproducing the same structure of the proof (the general case, the particular cases).
- It is important to note all the points that are constructed in GeoCoq's proofs. Sledgehammer does not construct these points (in general).
- In Rock, some tactics, such as Coplanar, can easily determine whether the statement to be proven is true or false. In Isabelle/HOL. Sledgehammer, which is more general, must be – sometimes – guided by the translator.
- If reading the GeoCoq code requires an intermediary: CoqIde, for example to understand the state of the proof at a given line, the Isar language is very fluid and direct, more natural.

Porting : non Rock and non Isabelle/HOL users

- Here the emphasis is mainly on the use of very general tools (Sledgehammer), not only dedicated to geometry. On the other hand, these general tools can quickly find links between statements. which is very useful in the case of synthetic geometry.
- There is no magic wand to find all proofs systematically but using Sledgehammer tools is a magic wand to quickly find a statement or proof or part of proof that correctly matches the hypotheses.
- Often most theorems, in the Euclidean plane, can be proven more quickly, by more classic automatic geometry tools. But porting teaches that one can sometimes reduce the number of hypotheses, make the context more general or attempt slight modifications in the proof.

3 applications of IsaGeoCoq

- QUARESMA, Pedro et SANTOS, Vanda. *Four Geometry Problems to Introduce Automated Deduction in Secondary Schools*. Electronic Proceedings in Theoretical Computer Science, EPTCS, 2022, vol. 354, p. 27-42. →
(MaSciProûve Vol. 2 Num. 3 (Juillet – Août – Septembre 2023, p 32-39 (in french)
http://masciprouve.cafr-msa2p.be/MaSciProuve_Vol02_Num03.pdf)
- Viviani's theorem. GEX/JGEX. *06_viviani.gex. Point interior triangle*.
(MaSciProûve Vol. 4 Num. 3 (Juillet - Août - Septembre 2025, p. 32–38. (in french)
http://masciprouve.cafr-msa2p.be/MaSciProuve_Vol04_Num03.pdf)
- Non-euclidean geometry (with tarski's axioms)
In IsaGeoCoq (dev-AFP) : Tarski_Non_Euclidean.thy, Tarski_Non_Euclidean_Aristotle.thy,
Tarski_Non_Euclidean_Archimedean.thy

WIP

- Port of Book 1 of Euclid of GeoCoq but IsaGeoCoq not used.
- Pictures with GCLC
- IsaGeoCoq → Geolsa ?

Thanks

Discussion

- About dimensionless
- Suppressions
- Interpretations
- Restriction
- Integration with existing contributions in the AFP