

Towards automatic detection of geometric difficulty of geometric problems

Zoltán Kovács¹ András Kerekes²

¹PHDL, Linz, Austria + RISC, Hagenberg, Austria

²Radnóti Miklós High School, Szeged, Hungary



ADG Conference, 1 August 2025
CADE Stuttgart, Germany

Abstract

We compare three different options to define geometric difficulty C_g of geometry problems.

By computing correlation between the achieved values and the algebraic complexity C_a , and, alternatively, ad-hoc human intuition C_h , we confirm that our former definition seems acceptable.

Why is this interesting?

- ▶ Ranking geometry theorems → Wos' Research Problem 31 (1988): *What properties can be identified to permit an automated reasoning program to find new and interesting theorems, as opposed to proving conjectured theorems?*
Also studied e.g. by Colton, Bundy and Walsh; Puzis, Gao and Sutcliffe; Gao, Goto and Cheng; Gao, Li and Cheng; and Quaresma, Graziani and Nicoletti.
- ▶ Abar and de Almeida: research among university students to learn their personal opinions on geometric difficulty, classifying complexity *before* and *after* attempting to actually prove the statements.
- ▶ Algebraic complexity may be different from human perception and geometric complexities → a good measure that works in general? Having such a measure would help to classify problem setting in classroom situations, exams, or mathematics contests.
- ▶ Generating new geometry problems → automated estimation of complexity.

Possible definitions of complexity

Algebraic, Z.K., T. Recio and M. P. Vélez, 2023

Let C be a geometric construction with hypotheses $H_1, H_2, \dots, H_r$ and thesis T . Prove the statement

$$H_1 \wedge H_2 \wedge \dots \wedge H_r \Rightarrow T$$

by considering an algebraic proof

$$1 = f_1 h_1 + f_2 h_2 + \dots + f_r h_r + f_{r+1} \cdot (zt - 1)$$

of the statement, where $h_1, h_2, \dots, h_r$ are algebraic translations of hypotheses $H_1, H_2, \dots, H_r$, and t is an algebraic translation of thesis T . Then, the *algebraic complexity* C_a of the statement is the maximal degree of f_r , that is

$$C_a := \max_{i=1}^{r+1} \deg f_i.$$

Possible definitions of complexity

Geometric, S. Bartha, A.K., Z.K. and T. Recio, 2024

Let C be a geometric construction with hypotheses $H_1, H_2, \dots, H_r$ and thesis T . Prove the statement

$$H_1 \wedge H_2 \wedge \dots \wedge H_r \Rightarrow T$$

by using the Geometric Deductive Database method (GDD) in Java Geometry Expert and consider the number of steps of the proofs, exported as a directed acyclic graph in GraphViz format. Then, the *geometric complexity* C_g of the statement is the number of steps (technically, the *number of lines*) in the output.

Possible definitions of complexity

Geometric (3 variations), A.K., Z.K., present contribution

Let C be a geometric construction with hypotheses $H_1, H_2, \dots, H_r$ and thesis T . Prove the statement

$$H_1 \wedge H_2 \wedge \dots \wedge H_r \Rightarrow T$$

by using the Geometric Deductive Database method (GDD) in Java Geometry Expert and consider the following:

- ▶ longest directed path (“Ldp”),
- ▶ number of vertices (“#v”),
- ▶ number of edges (“#e”)

of the obtained directed acyclic graph.

A simple example

Medians of a triangle intersect each other in one point, algebraic complexity (GD), $C_a = 2$

AK-23.ggb

File Edit View Options Tools Window Help

Sign in

CAS

```
47 s6:-v6+2*v12=0
   → s6 : 2 v12 - v6 = 0
48 s7:-v11+v13-v12*v13+v11*v14=0
   → s7 : v11 v14 - v12 v13 - v11 + v13 = 0
49 s8:v6*v7-v5*v8-v6*v13+v8*v13+v5*v14-v7*v14=0
   → s8 : -v13 v6 + v13 v8 + v14 v5 - v14 v7 - v14 v8 + v5 v14 = 0
50 s9:-1-v10*v13*v15+v9*v14*v15=0
   → s9 : -v10 v13 v15 + v14 v15 v9 - 1 = 0
51 Now we consider the following equation:
52 s1*(1/2*v14*v15-1/2*v15*v6)+s2*(-1/2*v13*v15+1/2*v14*v15)=0
   → 1 = 0
53 Contradiction! This proves the original statement.
54 The statement has a difficulty of degree 2.
55
```

Graphics

$F(v_{11}, v_{12})$

$$\begin{aligned} -v_1 - v_5 + 2v_{11} &= 0 \\ -v_5 - v_6 + 2v_{12} &= 0 \end{aligned}$$

$E(v_9, v_{10})$

$$\begin{aligned} -v_3 - v_5 + 2v_9 &= 0 \\ -v_4 - v_6 + 2v_{10} &= 0 \end{aligned}$$

$G(v_{13}, v_{14})$

$$\begin{aligned} -v_4 v_{11} + v_3 v_{12} + v_4 v_{13} - v_{12} v_{13} - v_3 v_{14} + v_{11} v_{14} &= 0 \\ v_6 v_7 - v_5 v_8 - v_6 v_{13} + v_8 v_{13} + v_5 v_{14} - v_7 v_{14} &= 0 \end{aligned}$$

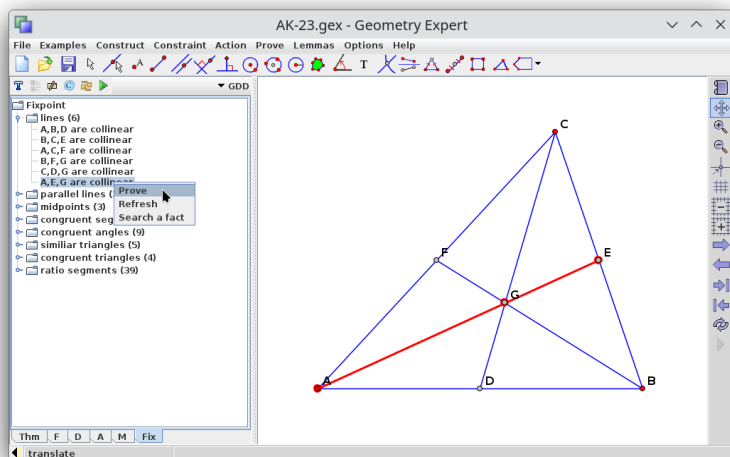
$D(v_7, v_8)$

$$\begin{aligned} -v_1 - v_3 + 2v_7 &= 0 \\ -v_2 - v_4 + 2v_8 &= 0 \end{aligned}

Input:$$

A simple example

Medians of a triangle intersect each other in one point (JGEX)



A simple example

Medians of a triangle intersect each other in one point, geometric complexities



$$C_g = 43$$

$$L_{dp} = 7$$

$$\#v = 13$$

$$\#e = 14$$

A simple example

Computing C_g by counting the lines of the GraphViz output for the GDD proof

```
AK-23.gv      [----] 1 L:[ 1+42 43/ 44] *(4312/4313b) 0010 0x00A
strict digraph
node [shape = box, color = black, style = filled];
edge [dir = back, tooltip = " "];
1 -> 2;
2 -> 3;
3 -> 4;
4 -> 5;
5 -> 7;
7 -> 10;
10 -> "F is the midpoint of AC";
10 -> "D is the midpoint of BA";
5 -> 8;
8 -> 9;
9 -> "E is the midpoint of CB";
9 -> "D is the midpoint of BA";
4 -> 6;
6 -> 9;
9 -> "E is the midpoint of CB";
9 -> "D is the midpoint of BA";
1 [ label = <1> G,F,A are collinear<BR/>
<FONT POINT-SIZE="10">Rule 1</FONT>>, tooltip = "If AB || BC, then A, B and C are collinear.", style = "filled", shape = in
2 [ label = <2> GE || AC>, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://git
3 [ label = <3> <[EGC] = <[AGD]<BR/>
<FONT POINT-SIZE="10">Rule 13</FONT>>, tooltip = "SSS (side-side-side) similarity.\nif two triangles have all three pairs
4 [ label = <4> ΔACG=ΔEDG, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://git
5 [ label = <5> AC=GD = DE=GC>, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://
6 [ label = <6> <[ACG] = <[EDG]<BR/>
<FONT POINT-SIZE="10">Rule 2</FONT>>, tooltip = "For two angles <[l1,l2], <[l3,l4]. If l1 = l3 and l2 || l4, then <[l1,l2]
7 [ label = <7> GC=FD = GD=BC<BR/>
<FONT POINT-SIZE="10">Rule 3</FONT>>, tooltip = "If AB || CD and E is the intersection of AC and BD, then EA / EC = EB / ED
8 [ label = <8> BC=DE = BE=AC<BR/>
<FONT POINT-SIZE="10">Rule 3</FONT>>, tooltip = "If AB || CD and E is the intersection of AC and BD, then EA / EC = EB / ED
9 [ label = <9> AC || DE<BR/>
<FONT POINT-SIZE="10">Rule 35</FONT>>, tooltip = "Middle Connection Theorem for Triangles.\nThe line connects the midpoint
10 [ label = <10> BC || DF<BR/>
<FONT POINT-SIZE="10">Rule 35</FONT>>, tooltip = "Middle Connection Theorem for Triangles.\nThe line connects the midpoint
"F is the midpoint of AC" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"E is the midpoint of CB" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"E is the midpoint of CB" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
```

A simple example

Computing C_g by counting the lines of the GraphViz output for the GDD proof

```
AK-23.gv [----] 1 L:[ 1+42 43/ 44] *(4312/4313b) 0010 0x00A
strict digraph "" {
node [shape = box, color = black, style = filled];
edge [dir = back, tooltip = " "];
1 -> 2;
2 -> 3;
3 -> 4;
4 -> 5;
5 -> 7;
7 -> 10;
10 -> "F is the midpoint of AC";
10 -> "D is the midpoint of BA";
5 -> 8;
8 -> 9;
9 -> "E is the midpoint of CB";
9 -> "D is the midpoint of BA";
4 -> 6;
6 -> 9;
9 -> "E is the midpoint of CB";
9 -> "D is the midpoint of BA";
1 [ label = <1> G,E,A are collinear<BR/>
<FONT POINT-SIZE="10">Rule 1</FONT>>, tooltip = "If AB || BC, then A, B and C are collinear.", style = "filled", shape = in
2 [ label = <2> GE || AG<, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://github
3 [ label = <3> <[EGC] = <[AGD]<BR/>
<FONT POINT-SIZE="10">Rule 33</FONT>>, tooltip = "SSS (side-side-side) similarity.\nIf two triangles have all three pairs
4 [ label = <4> ΔACG~ΔEDG<, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://git
5 [ label = <5> AC·GD = DE·GC<, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://
6 [ label = <6> <[ACG] = <[EDG]<BR/>
<FONT POINT-SIZE="10">Rule 2</FONT>>, tooltip = "For two angles <[l1,l2], <[l3,l4]. If l1 = l3 and l2 || l4, then <[l1,l2]
7 [ label = <7> GC·FD = GD·BC<BR/>
<FONT POINT-SIZE="10">Rule 3</FONT>>, tooltip = "If AB || CD and E is the intersection of AC and BD, then EA / EC = EB / ED
8 [ label = <8> BC·DE = BE·AC<BR/>
<FONT POINT-SIZE="10">Rule 3</FONT>>, tooltip = "If AB || CD and E is the intersection of AC and BD, then EA / EC = EB / ED
9 [ label = <9> AC || DE<BR/>
<FONT POINT-SIZE="10">Rule 35</FONT>>, tooltip = "Middle Connection Theorem for Triangles.\nThe line connects the midpoint
10 [ label = <10> BC || DF<BR/>
<FONT POINT-SIZE="10">Rule 35</FONT>>, tooltip = "Middle Connection Theorem for Triangles.\nThe line connects the midpoint
"F is the midpoint of AC" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"E is the midpoint of CB" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"E is the midpoint of CB" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
```

A simple example

Computing C_g by counting the lines of the GraphViz output for the GDD proof

```
AK-23.gv [----] 1 L:[ 1+42 43/ 44] *(4312/4313b) 0010 0x00A
strict digraph " " {
node [shape = box, color = black, style = filled];
edge [dir = back, tooltip = " "];
1 -> 2;
2 -> 3;
3 -> 4;
4 -> 5;
5 -> 7;
7 -> 10;
10 -> "F is the midpoint of AC";
10 -> "D is the midpoint of BA";
5 -> 8;
8 -> 9;
9 -> "E is the midpoint of CB";
9 -> "D is the midpoint of BA";
4 -> 6;
6 -> 9;
9 -> "E is the midpoint of CB";
9 -> "D is the midpoint of BA";
1 [ label = <1> G,E,A are collinear<BR/>
<FONT POINT-SIZE="10">Rule 1</FONT>>, tooltip = "If AB || BC, then A, B and C are collinear.", style = "filled", shape = in
2 [ label = <2> GE || AG>, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://githu
3 [ label = <3> ∠EGC = ∠AGD<BR/>
<FONT POINT-SIZE="10">Rule 33</FONT>>, tooltip = "SSS (side-side-side) similarity.\nIf two triangles have all three pairs o
4 [ label = <4> ΔACG~ΔEDG>, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://githu
5 [ label = <5> AC·GD = DE·GC>, tooltip = " ", style = "rounded, filled", shape = box, fillcolor = "#FFA0A0", URL="https://
6 [ label = <6> ∠ACG = ∠EDG<BR/>
<FONT POINT-SIZE="10">Rule 2</FONT>>, tooltip = "For two angles ∠[l1,l2], ∠[l3,l4]. If l1 = l3 and l2 || l4, then ∠[l1,l2] :
7 [ label = <7> GC·FD = GD·BC<BR/>
<FONT POINT-SIZE="10">Rule 3</FONT>>, tooltip = "If AB || CD and E is the intersection of AC and BD, then EA / EC = EB / ED
8 [ label = <8> BC·DE = BE·AC<BR/>
<FONT POINT-SIZE="10">Rule 3</FONT>>, tooltip = "If AB || CD and E is the intersection of AC and BD, then EA / EC = EB / ED
9 [ label = <9> AC || DE<BR/>
<FONT POINT-SIZE="10">Rule 35</FONT>>, tooltip = "Middle Connection Theorem for Triangles.\nThe line connects the midpoint:
10 [ label = <10> BC || DF<BR/>
<FONT POINT-SIZE="10">Rule 35</FONT>>, tooltip = "Middle Connection Theorem for Triangles.\nThe line connects the midpoint:
"F is the midpoint of AC" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"E is the midpoint of CB" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"E is the midpoint of CB" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
"D is the midpoint of BA" [ fillcolor = "#F7CAC9", shape = oval, style = "filled", tooltip = " " ];
```

Setting up a database of geometric constructions

Database of GeoGebra files, joint work with Soma Bartha, June 2024, Linz, Austria



Setting up a database of geometric constructions

Discussion, joint work with Tomás Recio, February 2025, Madrid, Spain



Setting up a database of geometric constructions

Joint work with S. Bartha and T. Recio, published in May 2025

SPRINGER NATURE Link

Login

Find a journal

Publish with us

Track your research

Search

Cart

[Home](#) > [Annals of Mathematics and Artificial Intelligence](#) > Article

Automated analysis of the difficulty of secondary school geometry theorems

Research | Published: 19 May 2025

(2025) [Cite this article](#)



[Annals of Mathematics and Artificial Intelligence](#)

[Aims and scope](#) →

[Submit manuscript](#) →

Soma Bartha, András Kerekes, Zoltán Kovács  & [Tomás Recio](#)

 54 Accesses [Explore all metrics](#) →

Part of a collection:

[S804: ADG 2024 – Formalization of Geometry and Reasoning](#)

Setting up a database of geometric constructions

Joint work with S. Bartha and T. Recio, published in May 2025

Abstract

We elaborate, as a benchmark, a list of geometry theorems that are regularly used in Hungarian high schools, in specialized tracks for mathematics. Then, these statements are implemented as GeoGebra files, and algebraically proven (with certification) in GeoGebra Discovery. Next, these files are imported in Java Geometry Expert (JGEX), and geometrically proven with the associated Geometry Deductive Database method. In both programs, through quite different approaches in each case, we define and implement an algorithmic measure of the difficulty of a geometric theorem. Then, we compute the corresponding measures on the selected list of statements, and we analyze the obtained collection of pairs of difficulty grades, finding a moderate positive correlation (0.51) between GeoGebra Discovery's algebraic, and JGEX's deductive difficulty formulations. A correlation that seems to support the adequacy of the chosen approximation in our on-going work to algorithmically establish some *ad-hoc* measure of the difficulty of geometric statements that could be reasonably coincidental with the appreciation of human expert users (e.g. teachers).

Keywords Automated theorem prover · Algebraic methods · Deductive Database · GeoGebra Discovery · Java Geometry Expert · Euclidean Geometry

Comparison of difficulty of 14 classroom problems

Theorem	C_a	C_g	C_h	Ldp	#v	#e
Thales' $\odot$ thm	4	13	2.1	2	4	3
Converse of Thales' $\odot$ thm	4	13	2.1	1	3	2
Midline thm	2	10		1	3	2
Intercept thm	7	8	3.4	1	2	1
Theorem of inscr. $\angle$ in a $\odot$	7		3.2	1	2	1
$\angle$ of cyclic quadrilaterals			3.2	1	2	1
Rel. betw. inscr. $\angle$ and $\angle$ at the $\odot$			3.2	2	5	4
Tangent-secant thm	6	16	3.8	3	5	4
Wallace-Simson thm	16	53	4.8	8	20	19
Euler's line	7	120	5.4	11	34	46
9-point $\odot$	18	94	5	5	23	32
Circumcenter of a $\triangle$	2	25		3	9	8
Barycenter of a $\triangle$	5	43		5	9	10
Apollonius' thm	7		3.2	5	12	11

Results of the questionnaire

C_h is taken as the average of the answers (line 13), stddev is shown in line 14

Matematikai tételek nehézsége (v2) (válaszok) előadashoz																								
Fájl Szerkesztés Nézet Beszúrás Formázás Adatok Eszközök Bővítmények Súgó																								
Menük																								
AB1																								
Form_Responses1																								
1	Időbélyeg	Teljes tétel és megfordítása	Pitagorasz-tétel és megfordítása	Héron-képlet	Háromszög egyenlősége	Nagobb szöggel szemben nagyobb oldal van	Ceva-tétel és megfordítása	Mérelsoz-tétel és megfordítása	A párhuzamos szelők és szelőszakaszok tétele	Szögössze-tétel	Derékszögű háromszög magasságtétele	Befogótétel	Kerület- és középponti szögek tétele	Húrnyújtástétel	Külöb pontból húzott érintőszakasz hossza egyenlő a pontból húzott szelőszakaszok mértani közepével	Méző hurok darabjainak szorzata egyenlő	Ptolemaiosz-tétel	Euler-egyenlet	Fuethach-kör	Wallace-Simson egyenes tétele	A háromszög trigonometrikus területképlete	Színusztétel	Koszinusztétel	
2	2025.02.12. 15:28:40	1	1	2	1	2			4	4	1	2	4	5	5			7			5	2	1	
3	2025.02.12. 15:30:08	1	1	3	1	1	?	?	1	1	1	1	1	1	1	?	?	3	3	?	1	1	1	
4	2025.02.12. 15:34:48	3	3	4	4	4	6	6	2	3	3	3	3	4	4	4	7	5	5	6	4	4	4	
5	2025.02.12. 15:37:44	1	1	3	2	2	4	4	4	4	4	3	3	2	3	4	4	5	5	5	4	4	4	
6	2025.02.12. 15:38:12	3	3	4	1	2	4	4	2	4	5	4	3	5	5	5	4	6	5	7	5	5	4	
7	2025.02.12. 15:56:32	2	2	3	3	5			4	4	3	3	3	3	6	6			6					
8	2025.02.12. 16:40:25	1	2	2	1	1	4	4	2	3	3	3	3	3	3	3	5	5	4	3	3	3	3	
9	2025.02.12. 17:30:27	2	2	4	1	1			2	2	1	1	3	2	2	2	2	2	4	5	3	2	3	3
10	2025.02.12. 18:22:31	3	3	4	2	4	6	6	5	5	4	5	5	5	5	5	6	6	5		5	5	5	
11	2025.02.12. 22:11:38	1	1	4	3	1	4	4	3	2	2	2	2	2	1	3	3	5	4	5	3	2	2	
12	2025.02.17. 17:35:23	5	4	5	4	5	6	7	8	3	3	4	5	3	2	2	9	8	8		5	5	5	
13	Átlag	2,09	2,09	3,45	2,09	2,55	4,86	5,00	3,36	3,18	2,73	2,82	3,18	3,18	3,36	3,78	5,00	5,40	5,00	4,83	3,70	3,40	3,20	
14	Szórás	1,24	1,00	0,89	1,16	1,56	0,99	1,20	1,87	1,11	1,29	1,19	0,99	1,34	1,67	1,31	2,12	1,36	1,26	1,46	1,35	1,36	1,40	
15																								

A.K.'s classmates

Some of them participated in filling in a questionnaire



Correlation computations

	C_g	C_h	Ldp	#v	#e
C_a	0.58	0.71	0.47	0.57	0.54
C_g		0.84	0.88	0.98	0.99
C_h			0.81	0.86	0.85
Ldp				0.94	0.90
#v					0.99

The correlation computation was performed via Pearson's correlation formula for samples,

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

where n is the sample size, and x_i and y_i are the individual sample points indexed with i , $1 \leq i \leq n$, moreover, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ is the mean for sample points $x_1, \dots, x_n$, the same for $\bar{y}$, respectively.

Conclusion and future work

- ▶ $C_g \sim C_a$ is the closest geometric complexity.
 - ▶ $\#v$ and $\#e$ are very close to C_g .
 - ▶ C_h is, surprisingly, between C_a and C_g .
 - ▶ Both the amount of theorems and the number of participants in the questionnaire are quite low. The research should be therefore repeated
 - ▶ in the same school, in different classes of young learners,
 - ▶ in other Hungarian schools, in special math program classes,
 - ▶ in other countries,
- to work on a larger dataset.
- ▶ C_a could be defined differently, e.g. by taking the maximal number of terms in the f_i in the formula

$$1 = f_1 h_1 + f_2 h_2 + \dots + f_r h_r + f_{r+1} \cdot (zt - 1).$$

THANK YOU!

References I



C. Abar and M. V. de Almeida.

Levels of estimation of the complexity of a geometric statement, Feb. 2025.

Presentation at I Workshop IAxEM, 20 February 2025, Universidad Nebrija, Madrid.



S. Bartha, A. Kerekes, Z. Kovács, and T. Recio.

Automated analysis of the difficulty of secondary school geometry theorems.

Annals of Mathematics and Artificial Intelligence, 2025.



S. Colton, A. Bundy, and T. Walsh.

On the notion of interestingness in automated mathematical discovery.

International Journal of Human-Computer Studies, 53(3):351–375, 2000.



E. R. Gansner and S. C. North.

An open graph visualization system and its applications to software engineering.

Software—Practice and Experience, 30(11):1203–1233, 2000.



H. Gao, Y. Goto, and J. Cheng.

Explicitly epistemic contraction by predicate abstraction in automated theorem finding:
A case study in nbgr set theory.

In N. T. Nguyen, editor, *Intelligent Information and Database Systems, 7th Asian Conference, ACIIDS 2015, Bali, Indonesia, March 23-25, 2015, Proceedings, Part I, Lecture Notes in Artificial Intelligence (Subseries of Lecture Notes in Computer Science), Vol. 9011*, pages 593–602, Cham, 2015. Springer.

References II



H. Gao, J. Li, and J. Cheng.

Measuring interestingness of theorems in automated theorem finding by forward reasoning: A case study in Tarski's geometry.

In *2018 IEEE SmartWorld, Ubiquitous Intelligence & Computing, Advanced & Trusted Computing, Scalable Computing & Communications, Cloud & Big Data Computing, Internet of People and Smart City Innovation (SmartWorld/SCALCOM/UIC/ATC/CBDCom/IOP/SCI)*, pages 168–173, 2018.



A. Kerekes and Z. Kovács.

Java Geometry Expert: Visualizing proofs as a directed acyclic graph, Feb. 2025.

Presentation at I Workshop IAxEM, 20 February 2025, Universidad Nebrija, Madrid.



Z. Kovács, T. Recio and M. P. Vélez.

Showing Proofs, Assessing Difficulty with GeoGebra Discovery.

Electronic Proceedings in Theoretical Computer Science, 398:43–52, 2024.



Z. Kovács.

Automated detection of interesting properties in regular polygons.

Mathematics in Computer Science, 14:727–755, 2020.



Y. Puzis, Y. Gao, and G. Sutcliffe.

Automated generation of interesting theorems.

In G. C. J. Sutcliffe, editor, *Proceedings of the Nineteenth International Florida Artificial Intelligence Research Society Conference (FLAIRS 2006)*, pages 49–54, 2006.

References III



P. Quaresma, P. Graziani, and S. M. Nicoletti.

Considerations on approaches and metrics in automated theorem generation/finding in geometry.

In Proceedings ADG 2023, 2023.



P. Todd.

A method for the automated discovery of angle theorems.

Electronic Proceedings in Theoretical Computer Science, (352):148–155, 2021.



L. Wos.

The problem of automated theorem finding.

Journal of Automated Reasoning, 10:137–138, 1993.



Z. Ye, S. C. Chou, and X. S. Gao.

An Introduction to Java Geometry Expert.

In Automated Deduction in Geometry, pages 189–195. Springer Science + Business Media, 2011.